Progress report of the APEC project: "A Collaborative Study on Innovations for Teaching and Learning Mathematics in Different Cultures among the APEC Member Economies"

APEC - TSUKUBA International Conference

"Innovative Teaching Mathematics through Lesson Study"

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APEC-TSUKUBA International Conference
"Innovative Teaching Mathematics through Lesson Study"
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Forward

This is the progress report on the APEC project "A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies". It included the result of APEC - Tsukuba International Conference.

At the third APEC Education Ministerial Meeting held on 29-30 April 2004 in Santiago, the ministers defined four priority areas for future network activities. "Stimulating Learning in Mathematics and Science" is one of the four priority areas. Based on this priority, the project "A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies" was approved by APEC Member Economies in August 2005. The project is managed by the Center for Research in Mathematics Education (CRME) in Khon Kaen University and the Center for Research on International Cooperation in Educational Development (CRICED) in University of Tsukuba. At the first stage, we held APEC - Tsukuba International Conference on 'Innovative Teaching Mathematics through Lesson Study', in January 15-20th in Tokyo. The aim was to share research questions and develop collaborative framework for the implementation of innovative scheme in teaching and learning of mathematics. For stimulating learning in mathematics and science, we focused on Lesson Study to develop good practices as a way of innovation. We used VTR for sharing lesson and tried to clarify Lesson Study method and good practice. Those two are major features through project which contributes improvement of education, especially mathematics and science, in members of economies.

Based on financial support of APEC project and University of Tsukuba, the project organized following meetings:

- Survey of Mathematical Literacy in OECD-PISA and the Lesson Study movement in Developing Countries: Improving the Quality of Education for Developing Numeracy on Education for All
- APEC Open symposium: "International Symposium on Innovative Teaching Mathematics through Lesson Study"
- APEC Specialist session: "International workshop on Innovative Teaching Mathematics through Lesson Study". Including school visits to know what Japanese Lesson Study is.

The conference has 226 participants and observers from 13 members of economies and 7 other countries. 13 members: Australia, Chile, China, Hong Kong, Indonesia, Japan, Korea, Malaysia, Philippines, Singapore, Thailand, USA and Vietnam (alphabetical orders).

Followings are targets and results of the Tokyo conference 'Innovative Teaching Mathematics through Lesson Study'

- In order to develop a research proposal and collaborative framework for the implementation of innovation scheme in teaching and learning mathematics;
  - We shared good practices as models of lessons and lesson studies for developing good practices in each members of economies.
  - We shared Japanese Lesson Study which is a recommended way of professional developments, implementing curriculum and developing good practices.
  - We elaborated the study paper format for Thailand session
    - Good practice is defined by outcome.
    - Present good practice as a result of Lesson Study.
    - Lesson VTR is necessary for sharing good practice.
    - Defined format of the paper and VTR.

Through the conference, we discussed the conditions of good practice and the meanings of lesson studies. The former is related with the way of view a lesson. The discussion on the meanings of lesson studies is very important at shearing processes. We can categorize presented lesson studies as follows;

First category is comparative studies on lesson studies by researchers focused on cultural-educational differences behind lessons. We learned from following economies reports; TIMSS video tape study: Lecture from Hong Kong, Learners Perspective Study; Lecture and Reports from Hong Kong, Korea and Philippines

Second category is lesson studies for developing good practices by teachers and math-educators. They are focused on improvement of the qualities of education: students learning, teaching approaches, developing subject matter, curriculum implementations and professional development. Following economies' reports focused on ideas for developing practice; Australia, Chile, China, Indonesia, Japan, Malaysia, Singapore, Thailand, USA and Vietnam.

Both categories are useful and very important for our research project, "A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies". In relation to our project, first perspective on comparative study contributed to know cultural differences and second perspective on improvement of the qualities of education contributed to share the idea of innovation. Based on these fruitful discussions, we recognized both perspectives are very important and for developing good practices, we preferred Japanese meanings of Lesson Study. Even if we focused on Lesson Study there are varieties of meanings and implementing processes depending on economies but we shared Lesson Study method that it done by teachers' group, elaborating lessons each other through observing lessons each others. Such as the cycle of 'plan (preparation), do (study lesson: observation), see (reflection)'

We preferred Lesson Study from following reasons: First, it is focused on

improvement of the qualities of education in general. It is a way of professional development in any subject. Secondly, it is well known methods in other economies and already adopted at some economies such as USA, Indonesia and Thailand. Especially in USA, there is reform movement based on Lesson Study (See 'Before it too late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century (2000)'). Thirdly, because Lesson Study have already been adopted in economies, we can share the good experience, knowing what the significant of Lesson Study and knowing how to manage the Lesson Study project in each other. For example, US Lesson Study project and Thailand project show the real meaning of Lesson Study by their given data. Fourthly, mathematics is a most easily subject to share the idea of teaching approach. Beginning from mathematics, we can influence the idea of Lesson Study to the other subjects such as science.

The 'APEC International Symposium on Innovation and Good Practice for Teaching and Learning Mathematics through Lesson Study' will be held in Khon Kaen, Thailand in June 14-17th, 2006. In that symposium, we expect contributions from members of economies as their result of lesson studies based on developed frame work in Tokyo meetings. Through these processes, we share the methods of Lesson Study and good practices with VTRs which are useful for innovation of mathematics education in each member economy. We are looking forward to meet each other again and to share the good practices of mathematics teaching with VTR which are developed through Lesson Study.

We would like to acknowledge supported and contributed governmental organizations and institutions. APEC project "A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies" is proposed from Thailand. The conference is organized by the University of Tsukuba with the organizing committee with support of APEC Project Overseers, co-organized by : Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT) and Japan International Cooperation Agency (JICA), supported by Ministry of Foreign Affairs of Japan (MOFA), Japan Society of Mathematical Education (JSME), Japan Society for Science Education (JSSE). At last part of this forward, we would like to note the names of Takeshi Miyakawa, Kazuhiro Aoyama and Kimiho Chino, researcher of CRICED, that we could not complete this progress report without their hard editorial works.

March 31, 2006

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GREETING ADDRESS

Yoichi Iwasaki
President of the University of Tsukuba

I would like to welcome you all sincerely. As the president of the University of Tsukuba, I would like to say a few words for the opening of the University of Tsukuba and Asian Pacific Economic Cooperation Conference on 'Innovative Teaching Mathematics through Lesson Study'.

This conference is following on the Third APEC Education Ministerial Meeting that was held in April 2004 in Chile, putting "Skills for the Coming Challenges" as its whole theme. "Stimulating Learning in Mathematics and Science" is one of the four priority areas in the theme. In this conference, we would like to make use of excellent research results in mathematics education for contributing to APEC activities regarding the priority area. University of Tsukuba has supported and extended the joint research with Khon Kaen University on 'A Collaborative Study on Innovations for Teaching and Learning Mathematics in Different Cultures among the APEC Member Economies', which is a project under Education Network of APEC Human Resources Development Working Group. Center for Research on International Cooperation in Educational Development (CRICED) in the University of Tsukuba is aiming at sharing academic research results on mathematics education among APEC economies through its various activities.

The University of Tsukuba has been accepting researchers and students from all over the world under the policy of "internationally opened university," ever since its foundation. We promote large scale collaborative international research and conduct active international exchange activities. We are in partnership with international bodies such as UNESCO and World Bank. We have established APEC Research Center within the university in order to actively engage in APEC activities.

The University of Tsukuba has a long standing tradition in teacher training since its former body, first Normal School, funded in 1873, and has achieved significant results in education research. Currently, the specialized research on classroom practice called "Lesson Study" has been the focus of international attention from all concerned. This "Lesson Study", I've heard, has originated from educational practices of teacher training of former body.

In addition the University of Tsukuba, we had engaged in the Japan – US collaborative research on mathematics problem solving in the 1980's, and also held the international conference of the Psychology of Mathematics Education in 1993, as well as the international conference on the use of technology in mathematics in 1998. We have played an active role in introducing Japanese research activities on the

mathematics education to the world.

In taking into account such a history of academic exchange and educational research and growing needs for international cooperation in education in developing countries under the theme of “Education for All”, University of Tsukuba has established the CRICED in 2002 and served as a core center under the network of MEXT, Ministry of Education Culture Sports Science and Technology, for international cooperation. We hope to try to contribute the developments of APEC economies based upon the educational research results from Japanese education practices.

Lastly, I would like to extend our gratitude to the supporters of the meeting, especially Dr. Chira Hongladarom from APEC Lead Shepherd who invited us to co-host this time as well as the co-organizers MEXT and JICA, as well as Ministry of Foreign Affaires and other academic organizations. We hope the conference would be a full opportunity for the participants from APEC economies.

That concludes my remarks. Thank you so much.

GREETING ADDRESS

Masayuki Inoue
Director-General for International Affairs, MEXT

Good morning ladies and gentlemen. My name is Masayuki Inoue, Director-General for "International affairs of the Ministry of Education, Culture, Sports, Science and Technology." This ministry has existed since 2001 as a result of the merger of ex-*Monbusho* and *ex*-Science and Technology agency, which was very timely for the future of education and science and technology.

Distinguished participants, ladies and gentlemen; I would like to express my sincere thanks to professor Iwasaki, the president of University of Tsukuba, and those lecturers who came a long way from various parts of the world.

It was two years ago in April 2004, in Santiago, Chile, that a Third APEC Education Ministerial Meeting was held. At the meeting, four priority areas were identified, as were "English and other foreign language learning", " Stimulating learning in Mathematics and Science", "Using ICT for teaching and learning", and "Governance and Systematic reform in education". Those countries concerned were to address those priorities in cooperation. This symposium is a follow up of the ministers' meeting. Japan has been a fine contributor in the area of math and science education, both in the APEC and a bilateral framework and has been cooperating with the Philippines, Indonesia and Cambodia for their improvement of such education.

As I said, in the APEC meeting, that ICT was listed one of the topics. Last week, together with his Excellency Kosaka, I attended the meeting in London, which was organized by the Department of Education and Skills of the UK government, and in which Ministerial level representatives from 60 countries participated. The audience was one hundred fifty members gathered in London. The potential of ICT was discussed actively during the meeting. I believe mathematics and science education, and ICT will become an increasingly important agenda for the future meetings of this nature.

There is a study called PISA hosted and conducted by OECD. Japan is ranked among top countries as to scientific literacy and mathematics literacy, according to the reports of a PISA study in 2000 and 2003.

On the other hand, students keep themselves away from science and mathematics education. This is probably the same as the case with the countries gathered here and is a very big problem. The 21st century is called "the century of knowledge economy," where we should entirely rely on people's knowledge and wisdom. In such an era, I think, avoidance of science and mathematics is a great disadvantage.

In this context, by studying the history and background of the development of the math and science education in Japan, how to address the problem we face today will be examined in this symposium, which will be highly meaningful to us. So, this, I believe, is the opportunity for everyone to share experiences, knowledge and to learn each other. And I'm convinced that this symposium will contribute greatly in making improvements in study on classroom and teaching methods of the mathmatics and science education in the APEC region.

Lastly, I would like to express my gratitude for the government of Thailand and University of Tsukuba. I would like to close my opening remarks. Thank you very much.

CONTENTS

GENERAL INTRODUCTION

At the third APEC Education Ministerial Meeting held on 29-30 April 2004 in Santiago, the ministers defined the four priority areas for future network activities. “Stimulating Learning in Mathematics and Science” is one of the four priority area. Based on this priority, the APEC project “A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies” was approved by APEC Member Economies in August 2005. The project is managed by the Center for Research in Mathematics Education (CRME) in Khon Kaen University and the Center for Research on International Cooperation in Educational Development (CRICED) in University of Tsukuba.

The project aims at: 1) to collaboratively develop innovations on teaching and learning mathematics in different cultures of the APEC Member Economies, and 2) to develop collaborative framework involving mathematics education among the APEC Member Economies. For these aims, the project focuses on the good practices in school classroom and ways of professional development such as the Lesson Study in each Member Economies. As the goal of project, we would like to publish the report (or book) with CD-roms including good teaching practices of participated economies and models of good practices which enable to use for the innovation of mathematics education in APEC economies and the world.

In order to achieve the goal of the project, we set two meetings within the four phases of the project:

Phase I, open symposium and closed workshop (specialist session) among key mathematics educators from the cosponsoring APEC Member Economies hosted by Center for Research on International Cooperation for Educational Development, University of Tsukuba, Japan will be organized in order to develop a research proposal and collaborative framework for the implementation of innovation scheme in teaching and learning mathematics (January 2006).

Phase II, each cosponsoring APEC Economy will develop some examples based on the framework (February-March 2006).

Phase III, the International Symposium will be organized in order to share and reflect on each Economy’s research results and best practice. The Symposium will be hosted by Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University, Thailand (May 2006).

Phase IV, The product for innovation of mathematics education will be developed. (July, 2006)

The project itself is carried out by the people invited in the two meetings at Phase I and Phase III. At the same time, to open the ideas for good practices to everyone in each society, the open symposium will be set in each meeting. We are expecting to invite same people in the both meetings. Depending on the restriction of the grant, we

are expecting to invite people from the following countries: USA, Japan, Korea, Australia, Chili, China, Hong Kong, Vietnam, Thailand, Philippines, Indonesia, Malaysia and Singapore. At the same time, we have been making effort to invite more people from other APEC economies. Participating of matheducators who have wishes to come to self fund will be acceptable.

The foci of the two meetings are to share the ideas of good practices from participants and structuring, developing and reviewing the product with VTR for teacher education and reform movement in Mathematics Education.

Ways of Publications

We develop our results through first and second meeting. At first meeting, participants present the discussion paper with some examples of good practice. At the end of first meeting, we would like to set editorial board, the clear format and structure of final papers with VTR. At second meeting in Thailand, participants are expecting to present papers with VTR based on the format and structure. After the discussion in Thailand meeting, we need to revise papers with VTR.

We are planning to report following ways. Contributions in the first meeting of Tokyo will be sited on the conference website and published from the Special Issues of Tsukuba Journal of Educational Study in Mathematics. Contributions in second meeting of Khon Kaen will be sited on the conference website and published from the Special Issues of Journal of Center for Research in Mathematics Education. Finally, we are planning to publish the comprehensive, revised and accepted papers in two meetings as a book with DVD or CD-roms.

Tentative Definition of Good Practice in Mathematics

We use the word of good practice in mathematics classroom as for the reform of each economy's mathematics education. It must not be the same depending on each economy. The ideas of innovation included in good practices might be useful for other economies. Depending on the result of TIMSS video tape study, we knew that there are differences among countries. There are strongly impressed people in the world by distributed VTR tapes. But we know that the study itself did not aim to know what the good practice is and which country should be the model of the mathematics lesson. In each economy there are a lot of researches in classroom and matheducators have been developing the good practices. On the other hands, there are problems that they can not write what, how and why good it is because there are no appropriate scientific ways to illustrate it as understandable among different cultural societies.

This project focus on gathering good practices themselves from participants in each economy and discuss what is good, why it is and how the teacher can develop such a good practice. If we discuss these points, we may know that how each good practice

has been done on different cultural setting and through the using these difference as our mirrors, we revalue our good practice from different perspective and get ideas and models for innovation of our mathematics education and try to use the model to the reform.

Indeed, TIMSS video tape study enables us to know how useful to look at a short part of the lesson and discuss about analyzing of it by each of us such as why it is good and why teacher did such a way. Through the meetings, we would like to talk about each good practice and define how we can express good practice. At the beginning of this project, we tentatively define the good practice in mathematics with following conditions.

1) It is visible, recordable in the classroom and can be showed to other people.
2) It may be known as a good approach in an economy.
3) There is a teacher who is well known by its approach.
4) It may be known as useful for the reform of mathematics education.
5) Many teachers may have their wish to do the same approach.
6) It may be taught in the teacher education (pre-service or in-service)
7) Against its approach, on contrast, there are different/traditional approaches based on different/traditional value.

These conditions are tentative as for imaging what it is. One of the goals of project is to develop visible models of good practices which can be used for teacher educations with DVD or CD-roms (or distributed through Internet) in each economies. Thus in the meetings, it is necessary to show and share examples by VTRs (digital movies).

Structure of Meeting in Tokyo, January 15-16, 2006

The aims of Phase I, first meetings, are constructed with two components to share the ideas for good (or best) practices, know the diversity meanings and approach in different cultures and share the good ideas of ongoing professional development such as the Lesson Study. First component of the meeting is open symposium based on key note lectures and symposium for shearing ideas. Second component is closed workshop (specialist session) to share the good practices in each economy, knowing why it is good and the developing shared frameworks for second meetings in Thailand.

The structure of meeting in Tokyo is explained in following schedule and obligation of participants of workshop will be written after the schedule.

Schedule of APEC - Tsukuba meetings in Tokyo

Following are schedule of APEC – Tsukuba meetings in Tokyo. Titles of lectures are

tentative.

Jan, 14 SAT. Arrival of Participants

Jan, 15 SUN. Open Symposium for planning collaboration (APEC participants survey of other countries' reform movements):

Improving the Quality of Education for Developing Numeracy on Education for All: Planning the International Collaboration for Future

Keynote lectures:

"Mathematical Literacy for Living from OECD-PISA perspective"

Dr,.Jan de Lange, Director, Freudenthal Institute, Netherlands.

Chair, OECD-PISA technical committee

"Japanese Lesson Study for Developing Best Practice"

Professor Akihiko Takahashi, DePaul University,

Professor Shizumi Shimizu, University of Tsukuba

Panel for sharing the ideas of projects for planning international collaboration on Numeracy:

"How have countries adopted the Lesson Study Approach for Educational Development on their JICA Projects ? "

General view of JICA Projects

Presentation from Countries on JICA Projects in Mathematics

Jan, 16 MON. Open symposium on APEC:

"International Symposium on Innovative Teaching Mathematics through Lesson Study"

Welcome Speech, Yoichi Iwasaki President of the University of Tsukuba

Opening Address, Dr. Chira Hongladrom, Lead Shepherd of APEC Human Resouce Development Working Group,

Key note lectures:

"Professional Development through Lesson Study: A Lesson learned from US"

Professor Catherine C. Lewis, Mills College USA

"Comparative Study of Mathematics Classroom"

Professor Frederick Leung, Hong Kong

Lectures:

"Innovation of mathematics teaching with ICT"

Professor Yasuyuki Iijima, Aichi University of Education, Japan

"Good Practice in Korea"

Professor Kyoungmee Park, Korea

"Open-ended Approach and Teacher Education"

Professor Maitree Inprashita, Thailand

General Discussion:

"Developing Research for Good Practice and its Methods"

Modulator: Professor Tad Watanabe, USA

Jan 16 to Jan 20. Specialist session on APEC

"International workshop on Innovative Teaching Mathematics through Lesson Study"

Jan 16. (after open symposium) Opening of Specialist Session

Jan 17. Morning School visit: Elementary School, University of Tsukuba.

Jan. 18. Morning School visit: Secondary School, University of Tsukuba

Jan. 20. Noon Closing

Ways of Specialist session, Jan 17 – Jan 20, in APEC-Tsukuba Conference

One of the goals of project is to develop visible models of good practices. At specialist session, we would like to discuss following points based on contribution from participants: what lesson study is, what the good practice is, why it is good, how it was developed. At the last stage of the session, we would like to conclude the reliable format, structure and categories of topics to describe good practice in mathematics education.

Specialist session will be managed by following two ways.

a. Lesson Study meetings at the attached schools on January 17 and 18 mornings

1) Short explanation of the lessons

2) Lesson observations

3) Discussion about lessons after observations

4) Discussion about what Japanese way of the lesson study is.

b. Presentation and discussion depending on categories

Presentation and discussion are categorized by topics depending on their full papers. Tentative categories are followings but it is not aimed to orient the contents of papers.

A. Lesson Study Project for special themes in mathematics education

B. Lesson Study Project for developing Innovative lesson approaches

C. Lesson Study Project for teacher education and professional development

E. Lesson Study Project for Implementing Curriculum

F. Lesson Study Project with ICT

Presentation and discussion are managed following ways.

1) Modulator and presenters are nominated depending on their papers

The role of modulator is refocusing on special topics to be discussed.

2) 30 minutes (or less) presentations including questions and answers

The aim of presentation is sharing ideas on good practices. In presentation, the presenter prepares his/her presentation with a short movie (10 minutes or less). Presentation must answer what is good, why it is good, and how it is developed. The methodology to show video movie, the way of presentation, itself is also meaningful for discussing how useful to look at a short part of the lesson for teacher education.

3) Modulator poses the questions and having short Break.

After three or four presentations, the modulator pose some questions which are useful for sharing key and meaning full ideas for innovation of mathematics education, knowing difference and developing the ideas for innovation of mathematics education.

4) Discussion in group depending on questions

Group discussion is done by participants from economies and observers.

5) Each group reports in 5 minutes

6) Modulator integrates reports and writes the concluding paper with reporters.

Venues of APEC - Tsukuba Conference in Tokyo

The meetings will be held following places:

Jan 15-16: International Conference Auditorium

JICA INSTITUTE FOR INTERNATIONAL COOPERATION

http://www.jica.go.jp/english/contact/ific/index.html

Jan 17-20: Attached Schools, University of Tsukuba at Tokyo

http://www.gakko.otsuka.tsukuba.ac.jp/map.jpg

and Meeting Rooms:

JICA INSTITUTE FOR INTERNATIONAL COOPERATION

Accommodations of representatives is going to be sited at the hotel in JICA Institute for International Cooperation Building in Tokyo

Important Information for Participants at First Meeting in Tokyo

Format of the papers

The format of all papers including lectures and presentation in open symposium is the PME format[1] by Adbe pdf or MS word. There are no limitation of pages in the case of lectures in January 15 and 16. In the case of lectures in January 15 and 16, simultaneous translation English-Japanese is set for Japanese participants. Thus, it is necessary to have papers for translation.

For specialist sessions, we are expecting 8 pages but if necessary we do not count the pages of Appendix. You can see any papers of PME on ERIC by the key words "Psychology of Mathematics Education"

Dead line of submission

Please send your paper to apec@criced.tsukuba.ac.jp **no later than January 7** for people can read before your coming to the conference.

Recommended format of the paper for specialist session

Followings are the expecting contents of full papers for specialist session:

- Description of Good Practices
- Why we can say it as good practices?
- What kind of reform is expected by such kinds of practices?

  If necessary:

  Please describe the setting in curriculum standard for explaining why it is good.

  Please explain it by the technical term of mother language as well as English meanings of it.

  Please explain it with relation to the world mathematics education research movement.

- Through the conference we would like to elaborate and develop appropriate ways of research paper format for describing and qualifying the good practice. Thus, you do not necessary to imagine the passed PME style research papers for writing your paper. It is better for us to consider using it to teacher

[1] http://www.pme30.cz/doc/PME30Template.rtf

education with example of VTR.

- If necessary, you can add pages with the appendix for describing details of the lesson but at the same time, people could not read if you put appendix too many protocols in the lesson. If necessary, it may be also useful if you can add the lesson plan with your economy's format as well as excerption of protocols of the lesson.

Additionally, at your presentation in conference, please use VTR or the data file of movie within 10 minutes for introducing a good practice. It may be necessary that VTR is edited with captions in English for understanding.

Following is an example:

http://e-archives.criced.tsukuba.ac.jp/en/result_data.php?idx_key=1034

You can see clearer/heavier version:

http://www.criced.tsukuba.ac.jp/math/teaching-material.html

Exploring Japanese Mathematics Lesson -For sharing key ideas-	.wmv (short ver.)(53.3MB)

This is an example. The ways of developing VTR and showing may be a good topic to be discussed in our meeting. In this case, it is developed for teacher education. It is expected that teacher educator, in this case Masami ISODA, explains each situations. Depending on the ways of using, it is not comfortable because there is no explanation about whole lesson and teaching plan. Captions are not protocols!

The video more than 10 minutes is not appropriate to understand because English is the second language for most of participants. It must be helpful for all people if your paper including some protocols and resume of VTR for understanding your VTR.

Supporting Travel Expense

Your travel expense will be supported from APEC Singapore Office or University of Tsukuba. The necessary information will be send to each person. If you have question in process, please ask immediately to the correspondences.

List of Correspondence of the Conference

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Japan International Cooperation Agency (JICA)

Supported by
Ministry of Foreign Affairs, Japan
Japan Society of Mathematical Education (JSME)
Japan Society of Science Education (JSSE)

Conclusion of Specialist Session of APEC Conference on Innovations in Teaching and Learning Mathematics Tokyo Session

Overall Purpose of the APEC Project

The project aims to (1) collaboratively develop innovations on teaching and learning mathematics in different cultures of the APEC Member Economies, and (2) develop a collaborative framework involving mathematics education among the APEC Member Economies.

Four Phases of the Project

Phase I A open symposium and closed workshop (specialist session) among key mathematics educators from the cosponsoring APEC Member Economies was hosted in January 2006 by the Center for Research on International Cooperation for Educational Development (CRICED), University of Tsukuba, Japan, in order to develop further a research proposal and collaborative framework for the implementation of innovation in teaching and learning mathematics.

Phase II Each cosponsoring APEC Economy will develop some examples based on the above collaborative framework (February to March 2006).

Phase III An International Symposium will be organized in order to share and reflect on each Economy's research results and best practice. The Symposium will be hosted by Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University, Thailand (June 2006).

Phase IV, The products for innovation in mathematics education will be developed and adopted in APEC economies (July 2006).

Products of the Project

The Products include Proceedings of the Tokyo Sessions and Proceedings of Khon Khaen Session. After the project, it is planned to develop a book on teacher development for good practice through Lesson Study with a VTR resources based on the products discussed below.

Focus of Tokyo meetings

There are several possibilities for innovation of mathematics education. Lesson Study which originated from Japan is currently a central focus in US and other economies for the professional development of teachers and the improvement of students' learning. The Tokyo meetings came to a consensus about the significance of focusing on Lesson Study as a means to innovation. Participants at the Tokyo meetings agreed that Lesson Study promotes good practices and these good teaching practices are powerful model for changing the quality of education. For enhancing Lesson Study in their economies, the Tokyo participants agreed to develop a VTR of good lessons as a

product of Lesson Study and to use it for teacher education.

Product of the Tokyo meetings

At the Tokyo meetings, researchers from different APEC economies presented research papers together with VTRs. In the specialist sessions, the main focuses of discussion were as follows: what is good practice, challenging to improve the quality of education through Lesson Study, and how to use a VTR resource for the aforementioned improvement. Good practice embodied in Lesson Study is based on outcomes of successful students' learning, including students' mathematical thinking, and can be used for further development or challenges. In conclusion, the Tokyo meetings developed a format for the final report which is to be used for teacher education in APEC economies. At the APEC Khon Kaen meeting, it is planned to produce the following components to support teacher education and professional development:

- **Research papers for developing good lesson.**
- **Videos with Lesson Plans**
- **Worksheets (as appendix) to accompany videos for Teacher Development**

Based on the results of APEC Khon Kaen meetings, we will publish a book consisting of reports and VTRs of Lesson Study from participating economies.

Necessary framework for developing products of the project

Through discussion at the Tokyo meetings, participants concluded that the following research topics are necessary for innovation of mathematics education through Lesson Study:

- What is Good Practice
  - Definition of desired mathematical performances by students
- What is Lesson Study
- Overcoming challenges that impede Good Practice
- Possible Themes
  - Lesson Study as professional development
  - Lesson Study as innovation / reform movement
  - Lesson Study to develop content
  - Lesson Study to develop teaching approaches
- Appendix of VTR

Core Ideas of Lesson Study to be used for the APEC Project

Key Principles for Adaptation of Japanese Lesson Study were identified at the Tokyo meetings. These are:

- Teachers helping teachers (teacher-led) to improve mathematics instruction in the classroom.
- Teachers play a central role in working with other teachers. Professors and researchers play supporting roles especially in providing theoretical framework.
- Decentralizing teacher development.
- Using actual classroom scenarios
- Adopting a Lesson Study cycle comprising planning → implementing and observing → discussing and reflecting (and the cycle repeats itself)
- Developing teacher knowledge through Lesson Study.

In the long run, grounded theories (or practical theories) are developed.

VTR form for good lesson to be used for teacher education.

- 10-minute video clip to illustrate the theme of the paper (e.g. lesson study to develop content)
- Explanation in the paper about how the video illustrates the theme
- VTR (by DVD-rom or CD-rom)
    - Title and others
        - Copyright and product data: including names of related people
        - Title of VTR (It does not need to be the Name of Topic)
        - Name of Topic, Grade, Name of teacher and school
    - Subtitles are necessary even in the case of English language
        - If possible, full translation is best
    - Phases in Lesson and understandable explanation about extract.
- Description (Appendix)
    - Title of VTR
    - Short summary of the lesson showing the aims of the lesson and the major problems or aspects covered in the lesson.
    - Components of the lesson and main events in the class.
    - Possible issues for discussion and reflection with in-service teachers or pre-service teachers observing the lesson.
    - Minimum information about copyright and acknowledgement of contributions of related people.

APEC Symposium

Innovative teaching mathematics through Lesson Study

16 January 2006

APEC Symposium

16 January 2006, PROGRAM

09:30 Opening Ceremony Chair: Masami Isoda (University of Tsukuba)

Yoichi Iwasaki, President of the University of Tsukuba

Masayuki Inoue, Director-General for International Affairs, MEXT

Chira Hongladarom, APEC HRDWG Lead Shepherd

10:00 Keynote Address Chair: Maxwell Stephens (Australia)

"Lesson Study in North America: Progress and Challenges"

Catherine C. Lewis (Mills College, USA)

11:00 Keynote Address Chair: Soledad A. Ulep (Philippines)

"Comparative Study of Mathematics Classrooms"

Frederick Leung (University of Hong Kong, Hong Kong)

12:00 Lunch Break

13:00 Lecture 1 Chair: Yeap Ban Har (Singapore)

"Innovation of mathematics teaching with ICT"

Yasuyuki Iijima (Aichi University of Education)

13:50 Lecture 2 Chair: Lim Chap Sam (Malaysia)

"Mathematics lessons in Korea"

Kyungmee Park (Hongik University, South Korea)

14:40 Break

15:00 Lecture 3 Chair: Grecia Galvez (Chile)

"Open-ended Approach and Teacher Education"

Maitree Inprashita, (Khon Kaen University, Thailand)

15:50 General Discussion

"Developing Research for Good Practice and its Methods"

Moderator: Tad Watanabe (The Pennsylvania State University, USA)

16:50 Closing Ceremony

Shizumi Shimizu (University of Tsukuba)

18:00 Reception

PROFESSIONAL DEVELOPMENT THROUGH LESSON STUDY: PROGRESS AND CHALLENGES IN THE U.S.[i]

Catherine Lewis and Rebecca Perry

Mills College, Oakland, California

This paper provides a brief history of lesson study in the United States, with a focus on areas of progress and challenge. Four areas of progress are identified: growth of interest among educators; growth of tools and resources; growth of understanding; and emerging evidence of effectiveness. Five challenges are identified: access to rich models of mathematical instruction; premature "expertise;" simplistic research models; limited opportunities for cross-site learning; and inadequate feedback loops linking lesson study to changes in curriculum and policy.

INTRODUCTION

Lesson study is the core form of professional development in Japan, and is often credited for the steady improvement of Japanese elementary instruction (Hashimoto, Tsubota, & Ikeda, 2003; Lewis & Tsuchida, 1997; Stigler & Hiebert, 1999). U.S. educators have shown enormous interest in lesson study since the Third International Mathematics and Science Study brought it to public attention in 1999; however, the U.S. has a history of educational faddism, in which many promising innovations have been discarded before being thoroughly understood or implemented (Burkhardt & Schoenfeld, 2003; Fullan, 2001). Will lesson study suffer a similar fate? This paper examines evidence of lesson study's progress and challenges in the U.S. to date.

LESSON STUDY'S PROGRESS IN THE UNITED STATES

Four areas of progress are identified: growth of interest in lesson study among U.S. educators; growth of tools and resources for lesson study; improved understanding of lesson study; and emerging evidence of lesson study's effectiveness in U.S. settings.

Growth of interest in lesson study.

In 1999, the Third International Mathematics and Science Study brought Makoto Yoshida's (1999) work on lesson study to a broad public audience (Stigler and Hiebert, 1999), provoking enormous interest in lesson study among US educators and researchers. Within three years, lesson study groups emerged in at least 200 U.S. schools across at least 25 states (Lesson Study Research Group, 2004a), and lesson study became the focus of dozens of conferences, reports and published articles in the US (e.g., Brown et al., 2002; Chokshi & Fernandez, 2004; Lewis 2002a,b; Lewis, Perry, & Hurd, 2004; National Research Council, 2002; North Central Regional Educational Laboratory, 2002; Richardson, 2004; Stepanek, 2001, 2003; Wang-Iverson & Yoshida, 2005; Watanabe, 2002; Wilms, 2003).

We are not aware of a systematic source of statistics on public lessons in the US, but we do that public research lessons now occur in many regions. For example, in the first half of 2005 alone, public lessons occurred in Olympia, Washington; Chicago, Illinois; Fresno, San Mateo, and Sonoma, California; several locations in and around Watertown, Massachusetts; and Des Moines, Iowa. At least five of these had more than 100 people in attendance.

Some interest in lesson study in the U.S. has come from quarters where there is not extensive lesson study in Japan, such as universities. U.S. interest in lesson study in the U.S. has emerged across grade levels (from preschool to university) and across subject areas, including science, mathematics, language arts, English as a second language, art education, social studies, special education, and no doubt other areas as well (Teaching American History, 2005; University of Wisconsin-LaCrosse, 2005).

Growth of tools and resources for lesson study.

Various tools for the pursuit of lesson study have been developed in the U.S., some based on Japanese practice (e.g, protocols for classroom observation and the post-lesson colloquium), and others in response to challenges that may be more prevalent in the U.S. than in Japan (e.g., how to get started with lesson study, how to develop collaborative norms within a lesson study group). Resources include individual protocols and agendas for parts of the lesson study process; handbooks; practitioner-oriented articles; and videos of lesson study in Japanese settings and in U.S. settings conducted by U.S. practitioners and by Japanese practitioners (Fernandez & Chokshi, 2002; Lesson Study Research Group, 2004b; Lewis, 2002b; Mills College Lesson Study Group, 2005, 2003a,b, 2000, 1999a,b; Wang-Iverson & Yoshida, 2005).

Improved understanding of lesson study.

Table 1 illustrates two alternative ideas about the mechanism by which lesson study improves instruction. We developed Table 1 as a foil for use in workshops, in response to the theory of lesson study that seemed to underlie questions often posed to us, such as "When do Japanese practitioners decide a lesson is good enough to be used widely?" and "If Japanese teachers spend so much time on one lesson, how do they ever get to all the lessons in the curriculum?" The view of lesson study labeled as hypothesis 1 – that it improves instruction primarily through the improvement of lesson plans – has characterized the early lesson work of some sites we have studied. For example, the teachers of Bay Area School District (BASD) initially used the phrase "Polishing the Stone" to describe their work, and originally planned to disseminate "polished" lesson plans on the district intranet as a primary outcome of their lesson study work. However, during their first year of work, BASD teacher-leaders began to redefine their work as teacher-led research on practice, and they began to regard the lesson plans as an inadequate representation of their learning from lesson study. As a result, they chose alternative methods to share their learning, such as open-house research lessons where visitors could participate in the whole process of lesson observation, data collection, and lesson discussion.

Emerging evidence of effectiveness of lesson study in U.S. settings.

When the senior author first gave talks about lesson study (in 1994), it was common for U.S. audience members to make comments like "lesson study is a good idea but it would never work in the U.S. because we are not a collaborative culture," or "Lesson study works in Japan because teachers know a lot of mathematics, but that's not true in the U.S." However, there are now emerging some "existence proofs" that U.S. teachers can use lesson study to *build* collaboration and content knowledge. The video of the U.S. lesson study cycle "How Many Seats?" illustrates how U.S. teachers can use lesson study to build both collaboration and content knowledge. In the segment of "How Many Seats?" excerpted in Table 2, Teacher 1 moves from confusion about the relationship of triangles and perimeter units ("tables" and "seats"; see problem in Table 3) to clear statement of the relationship between the two. Likewise, Teacher 5 gains insight into the physical reason for the numerical pattern. Solution and discussion of the problem to be presented to students and careful data collection during the research lesson support teachers' learning in these instances.

The teachers in "How Many Seats?" also build collaborative capacity, by setting norms for their work together, choosing one to monitor at each meeting, and sometimes changing their group operating procedures based on these discussions. For example, the group of teachers in "How Many Seats?" decides on a more active role for the (rotating) facilitator in confirming and marking group decisions, after monitoring of their norm "Sticking to the Process" reveals that some members are confused about the group's decisions. The following conversation occurs on Day 2 of the group's work, when group members are reflecting, at the end of the meeting, on the norm they chose to monitor that day: "sticking to the process." After one member comments that many ideas were discussed without a clear decision on them, another member suggests that the facilitator needs to take a stronger role.

> Teacher 6: I second what Teacher 3 says about, I think the facilitator's role is to stop, make sure you are on the process and make sure that everybody's, you know everybody's opinion is counted, you know.
>
> Teacher 5: hmm. So maybe we are hearing too that the facilitator needs to be a little bit more aggressive, a little bit you know more in there, saying let's slow down, let's poll everybody, let's say what we are doing right now. Would you feel more comfortable with that?
>
> (Nods, assents all around)
>
> The following day, when teacher 5 begins a segue into a new topic of conversation, the new rotating facilitator implements the more active role agreed upon the prior day: .
>
> Teacher 5: So this would be a good place for us to anticipate what we think is going to happen, misconceptions that might happen when they do 4, 5, and 6.

Teacher 1: Okay. But first let's hear from everybody I think, because we had kind of a proposal on the table and I think one of the things that happened yesterday was we would have a proposal and we sort of assumed everyone was on board, but we weren't. Is everybody on board with this? (Each member assents.)

This segment suggests that the group has actively used one of the tools provided (norm-setting and monitoring of norms), to create a more effective way of working together.

Other U.S. lesson study evidence suggests other types of teacher learning during lesson study. For example, the U.S. kindergarten teachers studied by Murata (2005) made connections between state standards and their own curriculum knowledge in the course of their lesson study work, shifting their view of the state standard in question from "no way" our students can do this to confidence that it can be mastered and knowledge about how go about it.

A technology-based "lesson-study inspired" innovation studied by Ermeling (2005) led U.S. high school science teachers to increase the student inquiry basis of their classroom lessons.

At one U.S. elementary school, teachers voted to practice lesson study on a school-wide basis in 2002, after volunteer groups of teachers found it to be useful, and this teacher-led lesson study has continued in every year since, growing from mathematics to include other subject areas at the instigation of the teachers. Table 4 shows the scale scores for the school on the state mathematics achievement test, along with those for the district and state as a whole. Over 2002-05, the three-year net increase in mathematics achievement for students who remained at this school was more than triple that for students who remained elsewhere in the district as a whole (90.5 scale score points compared to 25.8 points), a statistically significant difference ($F=.309$, $df=845$, $p<.001$). While a causal connection between the achievement results and lesson study cannot be inferred, other obvious explanations (such as changes in student populations served by the school and district) have been ruled out. School-wide lesson study appears to be a primary difference between the professional development at this school and other district schools during the years studied. [ii].

CHALLENGES TO LESSON STUDY IN THE UNITED STATES

Five areas of challenge have also emerged as lesson study has unfolded in the United States: access to rich models of mathematical instruction; premature "expertise" about lesson study; simplistic research models; limited opportunities for cross-site learning about lesson study; and inadequate feedback links between lesson study and changes in curriculum and policy.

Access to rich models of mathematical instruction.

Kyouzai kenkyuu (investigation of teaching materials) is a facet of lesson study that may enable teachers to deepen their understanding of mathematics, pedagogy, and student thinking (Hashimoto, Tsubota, & Ikeda, 2003; Takahashi et al., 2005).

Visiting Japanese educators often ask U.S. teachers how a particular topic is presented in the textbook, or suggest that U.S. teachers study a topic's presentation in several textbooks. This may be useful advice if the textbook's approach reveals interesting features of the topic. Unfortunately, this is not always the case. One group of mathematics coaches in California conducted a lesson study cycle on proportional reasoning. Accounts of Asian treatments of proportional reasoning provided some of the richest material for discussion (see Table 5, from Lo,Watanabe & Cai, 2000); in contrast, a U.S. textbook might provide few examples for teachers to deepen their thinking about the mathematics or pedagogy of proportional reasoning (see Table 6).

Premature "expertise."

Lesson study is a simple idea but a complex process. Even after a decade of studying lesson study in Japan, we are all still learning about lesson study's many forms and purposes. Remarkably, some U.S. trainers seem to believe that participation in one or two lesson study cycles qualifies them as lesson study experts who can provide definitive blueprints to others. Premature expertise may pose a substantial threat to lesson study, by generating a "been there, done that" attitude instead of a realistic expectation that "the road is created as we walk it together." [iii]

In contrast, a learning stance seems to characterize the work of settings such as BASD where lesson study has been sustained. During the first year of lesson study work, one of the BASD leaders answered a question about the attitudes essential to lesson study in the following way: .

> That you can always get better at teaching. That you're never at the end of the road…If you came into [lesson study] and you were [acting] like 'I'm the hottest thing out there and I've got all these great ideas and I'll share them with you guys'....you're not going to get anything out of it.

The expectation that teachers will learn about subject matter and its teaching-learning through lesson study has been a steady theme throughout the five years of the lesson study effort. For example, a video shot in 2002 and widely used to introduce BASD's lesson study work prominently features teachers' initial struggle to understand the mathematics of a problem and their strategies to build their own mathematical understanding (Mills College Lesson Study Group, 2005). In 2005, as one BASD lesson study group shifted its focus from mathematics to writing instruction, experienced teachers readily volunteered that they did not believe they had effective strategies for teaching writing. Two members commented afterwards on how lesson study fostered and was fostered by a culture in which "You're learning. You don't know everything. You're not busy hiding what you don't know."

Simplistic research models.

When we ask a roomful of U.S. educators to raise their hands if they have ever seen a promising innovation discarded before it has been thoroughly tried, virtually every hand in the room goes up. Simplistic research models may be one contributor to

premature innovation death. For example, lesson study might be regarded as something like aspirin, an easily transported treatment that interacts little with local site characteristics. Or lesson study may be regarded as a "recipe" that can be implemented at a site according to some fixed external instructions (perhaps with minor adjustments like one would make when using a recipe at high altitude).

Neither the metaphor of aspirin or recipe captures lesson study, because of the extensive interaction between lesson study and the local setting. What is needed to practice lesson study in a site where there is a coherent curriculum, tradition of collaboration, and history of careful study of student learning may be quite different from what is needed in sites where these do not exist. Lesson study might more appropriately be thought of as a system of learning with certain core principles, as sketched out in Table 1. Spreading a culture from one geographic location to another is perhaps the best analogy for lesson study; such cultural spread is something that can happen and has happened many times in human history. However, cultural spread is distinctly different from simply spreading the tools or recipes of a culture.

Limited opportunities for cross-site learning.

The United States is geographically large. Even though there are many lesson study efforts springing up, many U.S. teachers have little opportunity to experience lesson study outside of their own setting. To the extent that this is true, sites will reinvent the wheel, rather than learn from one another. For example, the idea of setting group norms and choosing one to monitor at each meeting, developed by teachers in one U.S. school district was eagerly embraced by others when they saw it in a workshop. Opportunities to see research lessons and post-lesson colloquiums conducted by teachers from other sites can provide an opportunity for immersion in another culture of lesson study, providing a vantage point on one's own assumptions, practices, and so forth.

Cross-national learning that includes educators from Japan may be a particularly potent form of cross-site learning, judging from U.S. teachers' reflections on cross-national workshops. Comments from U.S. teachers who engaged in cross-site lesson study with Japanese colleagues in August 2001 illustrate the kinds of reflection about lesson study and mathematics teaching-learning that may be stimulated by cross-site collaborative lesson study:

> [I learned that lesson study] is not so much about lesson planning as it is about research and watching children's learning
>
> I love the Japanese teachers' polite, validating comments to the students. "I don't require the correct answer."
>
> At the beginning of the week, I was more focused on the teacher. Now I can see and record students' mathematical thinking.
>
> There is no shortcut to doing the lesson planning and participating in lesson study yourself to become a helpful observer – DARN!

Effective observation involves skills, knowledge and preparation. This includes a "record of lesson" sheet, a copy of the lesson plan itself, and how effectively you can link teacher action to child's expression.

Create a need (hunger) for mathematical language; don't just give it to kids.

The blackboard is a record of the lesson. I often use the overhead (thus, erasing a lot) or erase what I've written on the blackboard due to lack of space. Mr. Takahashi's use of the blackboard has made me think of how I will use it in the future.

Inadequate feedback loops linking lesson study to changes in curriculum and policy.

In Japan there is an intimate relationship among lesson study, textbooks, and the national Course of Study. Advances in one arena tend to reshape the other arenas as well. For example, when Japanese elementary teachers used lesson study to try out lessons on solar energy (which was not then in the curriculum), this topic was picked up by other teachers, noticed by policymakers, and eventually became part of the national *Course of Study* (Lewis & Tsuchida, 1997). New elementary lessons are expected to prove themselves widely in public research lessons before finding their way into textbooks, and teacher-authors of textbooks are typically very active in lesson study, incorporating successful new approaches into textbook revisions.

MEXT (the Japanese Ministry of Education, Culture, Sports, Science and Technology) provides funding to schools across Japan that apply to be "designated research schools" for curricular innovations under consideration. Over a period of several years when an innovation is being considered or initiated, teachers at designated research schools engage in repeated cycles of lesson study, often inviting in university-based specialists and nationally known teachers interested in the particular innovation (Bjork, 2004; Lewis & Tsuchida 1997, 1998; Tam, 2004; Tsuneyoshi, 2001, 2004). Teachers at the designated research schools study existing curricula and materials (often including approaches from abroad), adapt or develop approaches they think will work in their own settings, and study students' responses to the new types of instruction. After cycles of internal lesson study, teachers conduct public research lessons that bring to life the local vision of the innovation, enabling visiting educators to observe the instructional approach and the students' learning and development, and providing a public forum for lively discussion of the local theory of the innovation. In this way, instruction, textbooks, and standards can evolve in tandem.

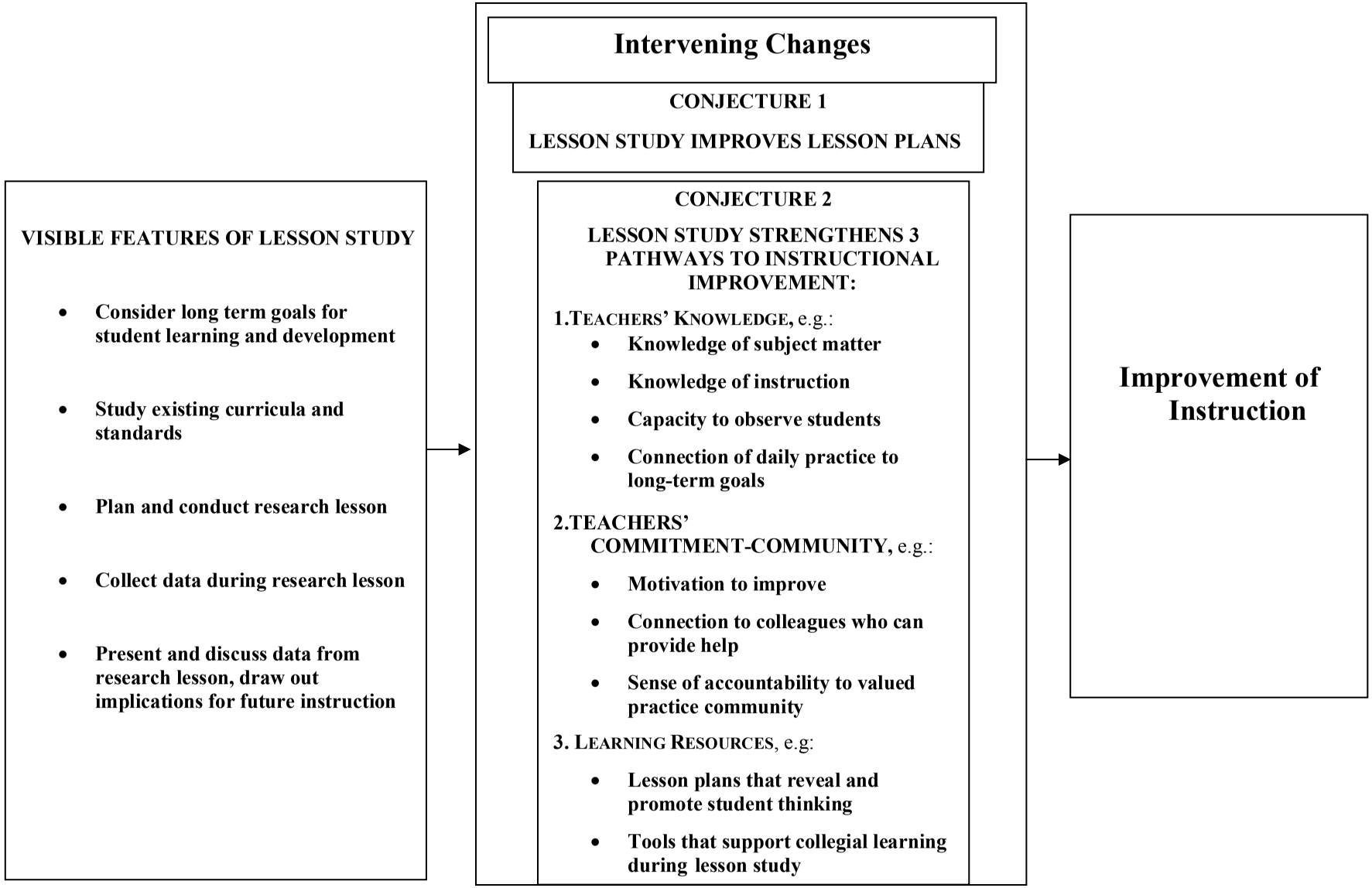
In contrast, the hard work of U.S. teachers to understand, for example, how a particular standard might be brought to life for first-graders (Murata, 2005) may remain within their group.. The major information conduits linking lesson study, textbooks, and educational policy in Japan are missing or sparse in the U.S.: for example, the well-known educators who travel to many lesson study sites to provide public commentary; the teacher-authors of textbooks who are heavily involved in lesson study; and the regional and national policymakers who attend research lessons and use

them as formative data on the strengths and shortcomings of policy and its implementation (Watanabe, 2002).

Conclusion.

In this international symposium, we have a valuable opportunity to find out whether the advances and challenges of lesson study experienced in the U.S. are similar to those found in other countries. We also have a valuable opportunity to share strategies for building progress and overcoming obstacles. As the Japanese say, "When three people gather you have a genius." I hope we can work with the great genius we have assembled here.

Table 1: How Lesson Study Results in Instructional Improvement: Two Conjectures



Date	Evidence	Researcher's Inference
8/7/02	Planning Meeting Teacher 1: I thought when we added a triangle we were adding two, but the output chart here is adding one, and I'm not, I don't understand why that is….. Teacher 6: Because the third one is now a combined one. Teacher 2: One plus two. It's plus two *this way* (moves finger horizontally across Teacher 1's chart, to show comparison between seats and tables). Teacher 1: Oh. Wait a second (studying triangles). Teacher 5: So maybe it would be a good time for us to do the activity? Teacher 1: (Laughing), yeah maybe! [teachers work problem with manipulatives and discuss]… Teacher 6: Because if you have one triangle you have three [sides], but then when you have two [triangles], one of those three [sides] becomes a combined. Teacher 1: *Two* of them become combined, that's why you don't have 5. Cause I'm thinking, how come I don't have 3 plus 2? Teacher 6: I just did the same thing! Teacher 4: You don't count the shared side. Teacher 5: It's the number of triangles plus two. Teacher 2: It's all plus two. It's plus two *this* way. [Gesturing across Teacher 1's chart, comparing triangles and perimeter units]… Teacher 1: But now why is that?… I'm still,	Teacher 1 is trying to understand the meaning of the "plus two" pattern in the chart. She initially merges the plus one pattern (each additional triangle adds one perimeter unit) and the plus two pattern (the number of perimeter units is two more than the number of triangles). Through trying different numbers with the manipulatives, she grasps the plus-two numerical pattern.

	though, why isn't it if I add a triangle…why am I not…[continues to work with the triangles, initially with puzzled tone of voice, then increasingly matter-of-fact as she tries different numbers of triangles] Three. So there's the two….[With confidence] This does not fit for zero triangles. This formula is not an n formula, it is not like "in any case" cause it has to fit for zero stage, right? Teacher 2: I don't know. I'd have to ask. Teacher 1: If the number of triangles is zero, you do not have two sides when you have no triangles.	
8/9/02	Planning Meeting Teacher 1: (Reading from group's instructional plan goals). Students will discover a pattern and they will represent the pattern as a rule. They will understand what a mathematical rule is and will be introduced to the idea of representing the rule as an equation. Teacher 2: So, representing the rule as an equation, that's a little bit.. Teacher 3: going in another direction Teacher 1: But it is an equation. We're saying: Number of tables plus two equals the number of …seats; that *is* where we want to get them to at the end of the easel time.	Now teacher 1 clearly describes the plus two pattern in her own words as she advocates for it in the lesson goals.
8/12/02	First teaching of research lesson: Teachers record the activities and speech of selected students, trying to create a complete record of what the selected student heard, saw, and did during the lesson.	
8/12/02	Colloquium of First Teaching Teacher 2: I noticed kids counting the seats different ways, and this was a kind of a big aha for me… When I've done the problem	Observation of student counting methods enabled Teacher 2 to understand the mathematics of the

	myself I've always counted [shows counting around the edge] and it didn't occur to me there was another way of counting it…But [student name] had laid out 20 triangles…and she was counting [demonstrates counting top and bottom alternately, followed by ends] and then it looked totally different to me; I could see there's 10 triangles on top, 10 on bottom, and a seat on either end. Now I was seeing the pattern a different way. Up until then, I had always seen it as you're taking away a seat and adding these two, taking away a seat and adding these two [shows adding a triangle and subtracting the side that is joined]. I was seeing a pattern from somebody else's perspective. That's why I thought it might be helpful to have kids talking about how they're counting it. How are you seeing the seats, and the numbers, and the increases, and where does that come from? So I think definitely having the kids use the manipulatives is important, and watching how they use them is going to tell us a lot about how did they see the pattern.	problem in a new way: that the two ends contribute the "plus two."

Table 2: Excerpts From The Lesson Study Cycle "How Many Seats?"

We have a long skinny room and triangle tables that we need to arrange in a row with their edges touching, as shown. Each side can hold one "seat," shown with a circle. Can patterns help us find an easy way to answer the question: How many seats fit around a row of triangle tables?

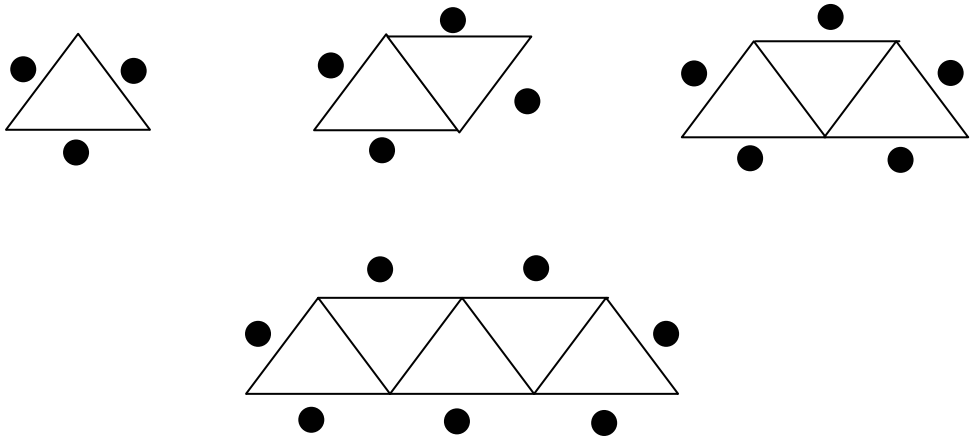


Table 3: Illustration of Problem Used In Lesson Study Cycle "How Many Seats?"

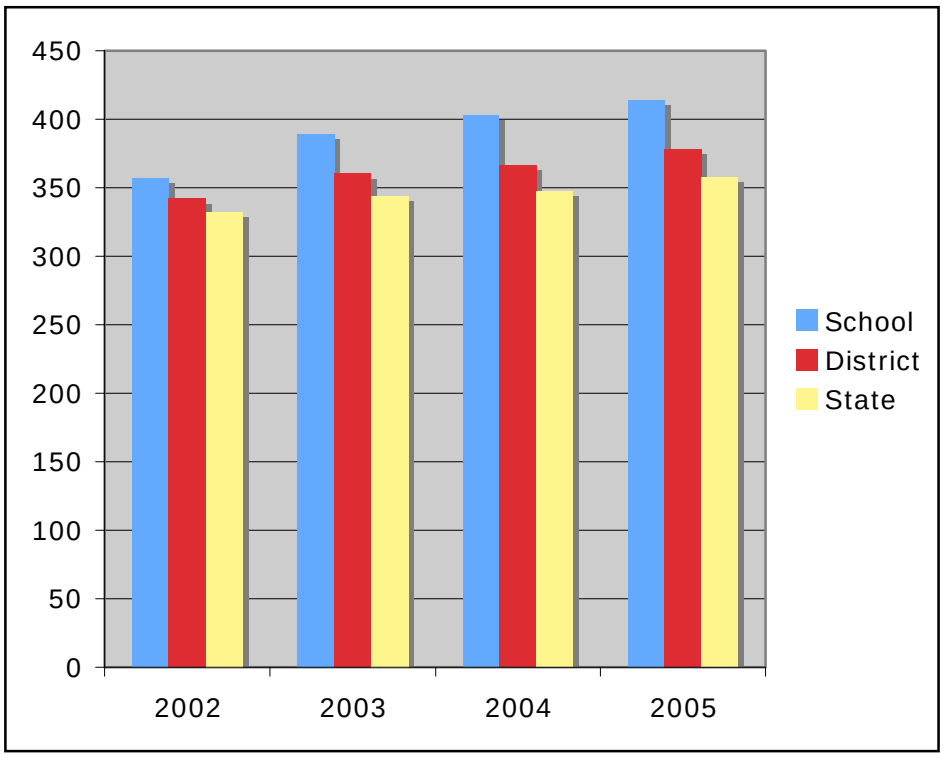


Table 4: California Standards Test in Mathematics: Mean Scale Scores, Grades 2-5

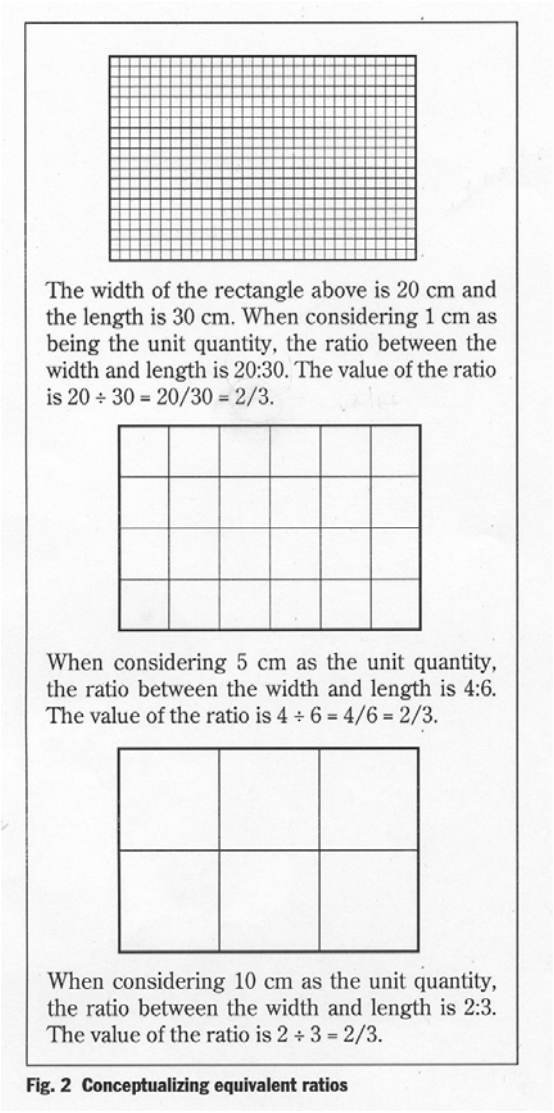


Fig. 2 Conceptualizing equivalent ratios

Table 5. Ideas about proportional reasoning introduced from research on Asian curricula (Lo, Watanabe, & Cai, 2004)

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[i] This material is based upon work supported by the National Science Foundation under Grant No. 0207259. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

[ii] To rule out competing hypotheses about causes of the increasing achievement, we identified other reform efforts that the school-wide lesson study school participated in during 2001-2005, and identified all other elementary schools (five) that participated these reform efforts. Gains in achievement for students who remained at each of these schools for longer than one year were compared with gains for all students who remained in the district. Only one school other than the lesson study school showed any statistically significant achievement gains relative to the district as a whole, and that school did not show sustained gains over three years. (The school that showed these gains was initially an Integrated Thematic Instruction school like Foothill, but the program was discontinued.)

[iii] From "Proverbios y Cantares, XXIX" by Antonio Machado, http://www.geocities.com/Athens/Delphi/5205/Proverbios_y_cantares.html; translator not given.

Comparative Study of Mathematics Classrooms

– What can be learned from the TIMSS 1999 Video Study[1]

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The Third International Mathematics and Science Study (TIMSS) 1999 Video Study aims at describing and comparing eighth-grade mathematics teaching practices among seven countries in order to identify similar or different classroom features. Since East Asian students have consistently performed well in recent international studies of mathematics achievement, this paper intends to analyze the TIMSS Video Study data for the East Asian country of Hong Kong in order to see whether there are classroom practices that can be used to explain students' high achievement in mathematics. The data analysis however yields conflicting results. While a qualitative analysis of the data shows that the quality of mathematics teaching in Hong Kong is high, a quantitative analysis of the same data shows that teaching in Hong Kong is rather traditional and teacher-centred. The conflicting results point to the complexity in interpreting video data on classroom practices and of achievement data in international studies. The results are then interpreted with respect to the underlying cultural values in East Asia, and implications for methodology in analyzing video data, as well as for educational reform in East Asian countries and other countries are discussed.

Introduction

Students from East Asian countries [2] have consistently outperformed their counterparts in the West in international comparative studies of mathematics achievement such as the *Third International Mathematics and Science Study* (TIMSS[3]) (Beaton *et al,* 1996; Mullis *et al,* 1997; Mullis *et al,* 2000; Mullis *et al*, 2004) and the OECD *Program for International Student Assessment* (PISA) (OECD, 2001; 2003; 2004). However, the high achievements of East Asian students do not seem to have been accompanied by correspondingly positive attitudes towards mathematics (Leung, 2002). An obvious question to ask of such

[1] Paper to be delivered at the APEC-Tsukuba Conference, Tokyo, Japan, 16 January, 2006.

[2] East Asian "countries" in this paper refer to Chinese Taipei, Hong Kong, Japan, Korea and Singapore. Although some of them (e.g. Hong Kong) are not countries, for convenience the generic term "countries" will be used to refer to all participants in these international studies.

[3] TIMSS was renamed Trends in *International Mathematics and Science Study* starting from the 2003 Study.

international studies is what accounts for high achievement, and in particular, what accounts for the high achievement of East Asian students despite their negative attitudes towards mathematics. Since students learn most of their knowledge in the classroom, it is reasonable to expect that the instruction they receive should be a major factor in influencing their achievement.

In this paper, the TIMSS 1999 Video Study data for the East Asian country of Hong Kong are analyzed to see whether there are classroom practices that can be used to explain students' high achievement in mathematics. Methodological issues related to comparative classroom studies are then discussed, and results of the Study are interpreted with reference to the East Asian culture. Finally some implications of the findings of the study are drawn for mathematics curriculum development in East Asian and other countries.

The TIMSS 1999 Video Study

The TIMSS 1999 Video Study (hereafter referred to as the Study) examined instructional practices in eighth grade mathematics for seven countries: Australia, Czech Republic, Hong Kong SAR, Japan, Netherlands, Switzerland, United States. The goals of the Study were to:

- describe and compare eighth-grade mathematics teaching across seven countries
- discover alternative ways to teach mathematics
- examine teaching in one's own country with fresh eyes, and
- create digital library of public use videos for teacher professional development

(Hiebert *et al*, 2003)

Japan did not collect video data for mathematics in 1999, but the Japanese data for the TIMSS 1995 video study were re-analyzed using the 1999 methodology in some of the analyses. For this reason, only the Hong Kong data will be highlighted for discussion below, since it is the only East Asian country for which data was collected in the 1999 Study.

Sampling, Data Collection and Analysis

Sampling

To obtain a representative sample of eighth-grade mathematics classrooms in each of the participating countries, a national probability sample[4] of a target of 100 schools was drawn in the Study. One mathematics class was then randomly selected from each of the schools, and only one lesson was videotaped for the sampled class. Including the 50 Japanese lessons videotaped in 1995, altogether 638 lessons were videotaped, ranging from 78 lessons (the Netherlands) to 140 lessons (Switzerland[5]) per country.

Since the eighth grade mathematics curriculum in the seven countries differs from each other, it has not been possible to match the content of the videotaped lessons in different countries. Instead, lessons were randomly selected across the school year so that they covered the content taught in the whole of the eighth grade in the country.

Data Coding and Analysis

Videotaping in all countries followed standardized camera procedures. Two cameras were used, with one camera focusing on the teacher and her interaction with students, and the other camera focusing on the whole class. All data from the seven countries were assembled together and analyzed by an international video coding team, advised by an expert group with members (known as national research coordinators) from each of the participating countries. Although the working language of the project was English, data analysis for individual countries was performed in the language used in the classrooms. Members of the international video coding team were all fluently bilingual (in the language used in the classrooms concerned and English) researchers, and working together they developed codes to apply to the video data. Three marks (i.e., the in-point, out-point, and category) for the codes were evaluated and included in the measures of reliability. For any code, if the reliability measures fell below the minimum acceptable standard after numerous attempts, it would then be dropped from the study. Altogether, 45 codes survived and were applied in seven coding passes to each of the videotaped lessons.

The Mathematics Quality Analysis Group

The quantitative analysis described above is fine grained, and allows details of the lessons to be captured. On the other hand, there is a danger that fine grained analysis would break down the lessons into minute constituent parts but the parts

[4] The sample was a Probability Proportional to Size (PPS) one, i.e., the probability that a school being chosen is proportional to the size of the school as measured by the number of eighth grade students in the school.

[5] In Switzerland, since there were three major languages of instruction, more schools were selected so that instructions across different language groups may be compared.

may not fit with each other to form back a meaningful picture of the lesson. For this reason, in addition to the quantitative analysis described above, a number of more qualitative analyses were performed. One such analysis was performed by an expert panel, known as the Mathematics Quality Analysis Group, comprising mathematicians and mathematics educators at the post-secondary level. The group reviewed a randomly selected subset of 120 lessons (20 lessons from each country except Japan[6]) and evaluated the quality of the lessons based on expanded "lesson tables" prepared by the international video coding team. The "lesson tables" contained detailed written descriptions of the lessons, including the classroom interaction, the nature of the mathematical problems worked on, goal statements, lesson summaries, and other relevant information. These lesson descriptions were examined "country-blind", with all indicators that might reveal the country removed.

Mathematics Classrooms in Hong Kong

A. Instructional Practices as Portrayed by the Analysis of the Codes

Whole-class interaction dominated

In describing the kinds of teacher and students interaction in the seven countries, the Study defined five types of classroom interaction: public interaction, private interaction, student presents information, teacher presents information, and mixed private and public work. An analysis of the different types of interaction showed that the Hong Kong classroom was dominated by public or whole-class interaction. Three quarters of the lesson time was spent in public interaction while 20% of the lesson time was spent in private interaction (see Table 1 below). These represent the largest proportion of lesson time in public interaction and the smallest proportion of lesson time in private interaction among the seven countries. As the Study Report commented, "Comparing across countries, eighth-grade mathematics lessons in Hong Kong SAR spent a greater percentage of lesson time in public interaction (75 percent) than those in the other countries, except the United States." (Hiebert *et al*, 2003: 54-55).

[6] Since this same group of experts performed a similar analysis on the 1995 TIMSS Video data, which included the Japanese data, the 1999 Japanese data was not included in this analysis.

Table 1: Average Percentage of Lesson Time Devoted to Public and Private Interactions

Country	Public interaction	Private interaction	Others
Australia	52	48	0
Czech Republic	61	21	18
Hong Kong	75	20	5
Japan	63	34	3
Netherlands	44	55	1
Switzerland	54	44	1
United States	67	32	1

Teacher talked most of the time

What were the Hong Kong teachers and students doing during the whole-class interaction time? The Study recorded and calculated the number of words spoken by the teachers and the students in the lessons[7] as indication of the kind of interaction that took place. As can be seen from Figure 1 below, Hong Kong teachers spoke an average of about 5800 words per lesson while their students spoke only an average of 640 words. Compared to other countries in the study, Hong Kong teachers (together with the US teachers who spoke an average of about 5900 words per lesson) were the most talkative among the teachers in the participating countries. In contrast, Hong Kong students were the least talkative among the students in all the seven countries.

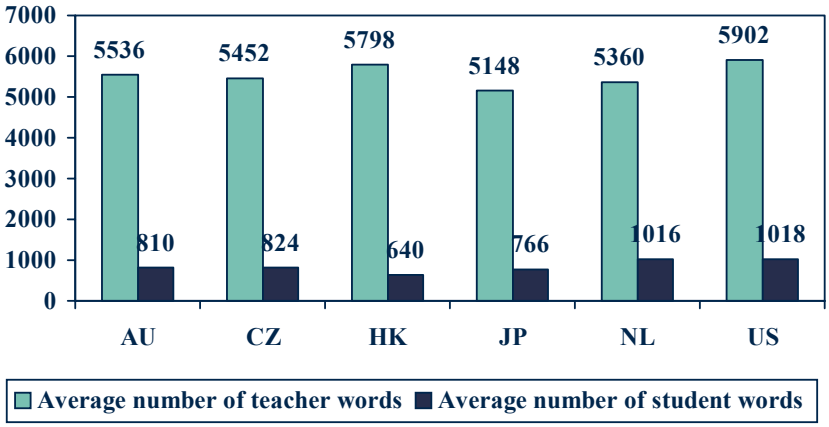


Figure 1: Average Number of Teacher and Student Words Per Lesson

Combining the two sets of figures in Figure 1, Hong Kong classrooms have the highest ratio of average number of words spoken by the teacher to those spoken by

[7] Since lesson duration varies across countries, the lesson time reported here is standardized to 50 minutes.

their students (Figure 2). As the Study Report pointed out, "Hong Kong SAR eighth-grade mathematics teachers spoke significantly more words relative to their students (16:1) than did teachers in Australia (9:1), the Czech Republic (9:1), and the United States (8:1)" (Hiebert *et al*, 2003: 109). When we factor in the relatively large class size of the Hong Kong classroom[8], the reticence of the East Asian students is even more striking.

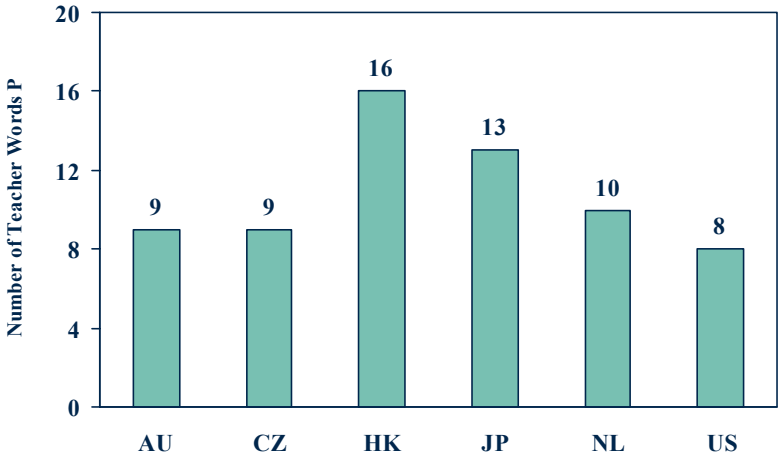


Figure 2: Average Number of Teacher Words to Every One Student Word Per Lesson

Students solved procedural problems unrelated to real-life following prescribed methods

In the Study, it was found that the lesson time in all the seven countries was dominated by students working on mathematical problems, and thus one of the major units of analysis in the study was the mathematical problems. Different aspects of the characteristics of the problems worked on in the lessons were coded for analysis, and results of some of the analyses are discussed below.

Nature of problem statements

One important characteristic of the mathematical problems is the nature of the problem statements. Three types of problem statements were defined in the Study

[8] The average class size of the lessons videotaped in Hong Kong was 37, which is significantly bigger than the class size of other countries in the study (except for Japan for which the class size data was not available) - the average class size in the other five countries ranged from 19 (Switzerland) to 27 (Australia). So a ratio between teacher words and student words of 16 to 1 in Hong Kong is in effect a ratio of nearly 600:1 as far as an individual student in concerned.

based on the kind of mathematical processes implied by the statements. They are using procedures, stating concepts, and making connections (Hiebert *et al*, 2003: 98). Figure 3 below shows the average percentage of problems of each problem statement type in the participating countries[9].

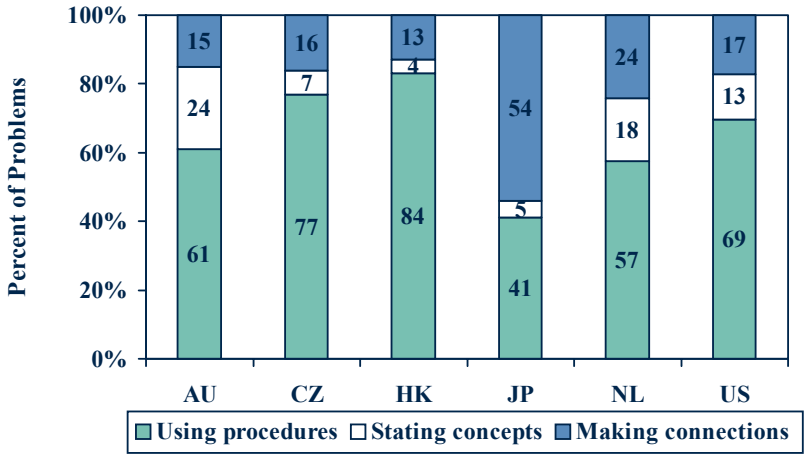


Figure 3: Average Percentage of Problems Per Lesson of Each Problem Statement Type

As can be seen from Figure 3, the problem statements of nearly 85% of the problems worked on in the Hong Kong classrooms suggest that they were typically solved by applying a procedure or a set of procedures. This percentage is highest among all the countries in the Study. Problems with statements that called for mathematical concepts or constructing relationships among mathematical ideas and facts were relatively rare. As the Study Report noted, "Hong Kong SAR lessons contained a larger percentage of problem statements classified as using procedures (84 percent) than all the other countries except the Czech Republic (77 percent)" (Hiebert *et al*, 2003: 98).

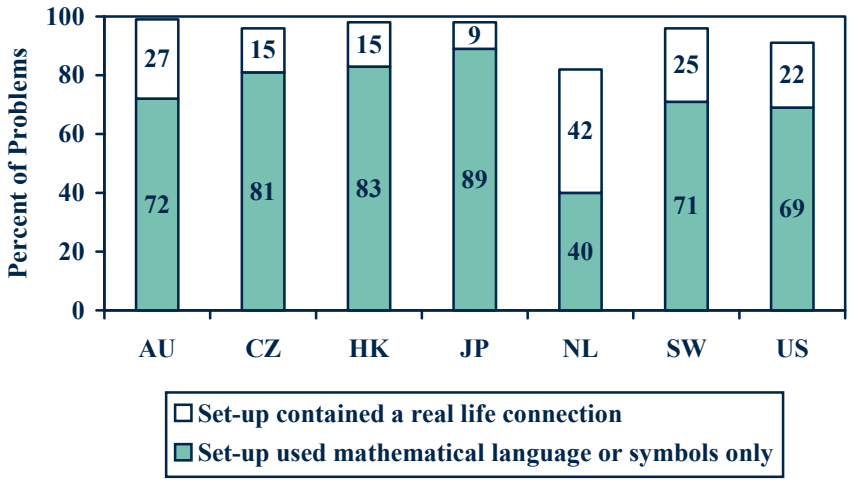
Contexts of the problems

In what contexts were these procedural problems set up when they were presented to the Hong Kong students? Mathematics problems are usually either set up within some real-life contexts or simply presented using mathematical language or symbols (e.g., Solve the equation: $x^2 + 3x - 8 = 0$). Many mathematics educators argue that mathematics problems presented within real-life contexts make mathematics more meaningful and hence more interesting for students.

[9] The data from Switzerland was not available since English transcripts were not available for all Swiss lessons.

Figure 4 below shows the average percentage of problems that were set up with a real-life connection compared to those that were presented with mathematical language or symbols only. As can be seen from Figure 4, Hong Kong lessons had most of the problems set up using mathematical language or symbols only, second to Japan. Only 15% of the problems had a real-life connection, and more than 80% of the problems were formulated with mathematical language and symbols only.

Figure 4: Average Percentage of Problems Per Lesson Set Up With a Real Life Connection or



With Mathematical Language or Symbols Only

Choice of solution methods

When Hong Kong students were presented with these procedural problems set up with mathematical language and symbols, how were they expected to deal with the problems? Were they expected to solve the problems with prescribed methods, or were they given a choice and encouraged to solve the problems using different methods? Mathematics educators usually think that to enhance students' problem solving ability, they should be encouraged to solve the same problem with different methods. In the Study, the number of problems worked on in which students had a choice of solution methods was noted, and the results are show in Table 2 below. In Table 2, the left hand column gives the average percentage of problems per lesson in which students had a choice of solution methods, and the right hand column shows the percentage of lessons where there were at least one problem worked on in which students had a choice of solution methods.

Table 2: Average Percentage of Problems Per Lesson and Percentage of Lessons With at Least One Problem in Which Students Had a Choice of Solution Methods

Country[10]	Average percent of problems with a choice of solution methods	Percent of lessons with at least one problem with a choice of solution methods
Australia	8	25
Czech Republic	4	20
Hong Kong	3	17
Japan	15	31
Switzerland	7	24
United States	9	45

It can be seen from Table 2 that compared with other countries, Hong Kong had the least amount of problems where students were given a choice of solution methods, whether measured by average percentage of problems per lesson or by the percentage of lessons with at least one problem in which students had a choice of solution methods. In only three percent of the problems worked on were students given a choice of solution methods, and such occasions happened in less than 20% of the lessons recorded.

So we can see from the three characteristics of the problems discussed above that the mathematical problems Hong Kong students worked on in their classrooms were mainly problems unrelated to real-life. The statements of the problems suggest that they were typically solved by applying a procedure or a set of procedures rather than calling for mathematical concepts or constructing relationships among mathematical ideas and facts. Furthermore, students were expected to follow prescribed methods in solving these problems instead of being given a choice of solution methods.

Summary

From the results presented above, the instructional practices in the Hong Kong mathematics classroom as portrayed by the analysis of the codes in the Study can be characterized as follows:

[10] For the Netherlands, there were too few cases reported and so the data was not shown here because the reporting standard was not met.

Whole-class interaction dominated the lesson time. During the whole-class interaction, the teacher talked most of the time while the students remained relatively reticent. The mathematics problems that students worked on during the lesson were mainly set up using purely mathematical language and symbols, and in contexts unrelated to real-life. These problems were also typically solved by applying a procedure or a set of procedures, following standard methods prescribed by the teacher.

From the viewpoint of most mathematics educators, the picture portrayed above is a mathematics classroom that is not very conducive to quality teaching and learning!

B. Quality of Content as judged by the Mathematics Quality Analysis Group

As described earlier, one of the qualitative analyses of the data in the Study was performed by an expert panel comprising mathematicians and mathematics educators. Panel members reviewed detailed descriptions of a random sub-sample of the videotaped lessons country-blind and made qualitative judgements about them. In addition to judging the content level of the lessons, the panel also assessed the quality of the mathematics in the lessons along four dimensions: *coherence, presentation, engagement* and *overall quality*. The results of the judgements of the Mathematics Quality Analysis Group are presented below.

More advanced content

The panel made judgement on how advanced the mathematics content in the lessons was, and placed each lesson in the sub-sample into one of five "curricular levels", from elementary (1) to advanced (5). The results of their judgement are shown in Figure 5 below. As can be seen from Figure 5, the panel found the content covered in the Hong Kong (and Czech Republic) classrooms relatively more advanced. The mathematics content of 20% of the lessons was judged to be advanced, while the content in none of the lessons was judged to be elementary. This is in great contrast to other countries in the Study, where three of them (Australia, the Netherlands and the United States) did not have any lessons with content judged to be advanced (and Switzerland had only 5% of the lessons with content judged to be advanced), and the mathematics content in at least 10% of the lessons in four countries was judged to be elementary.

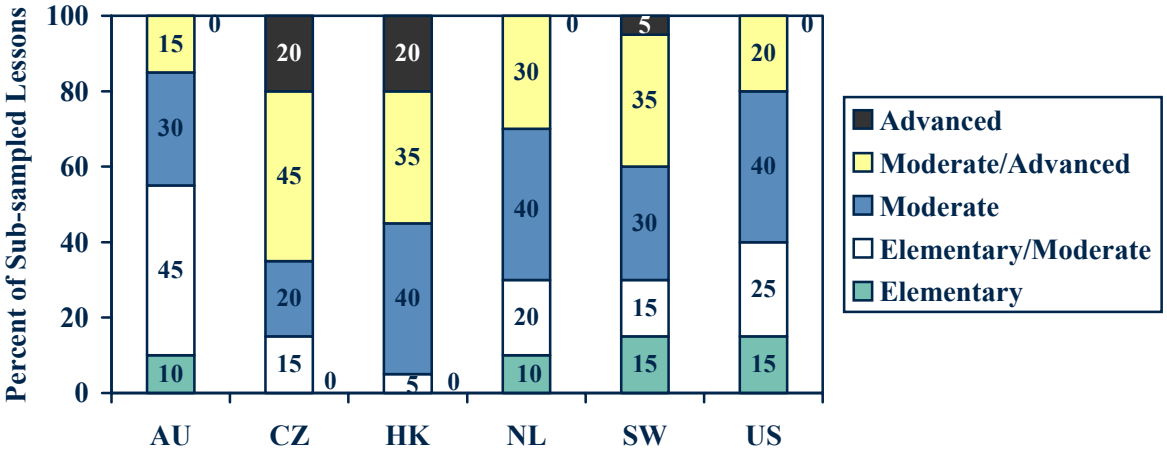


Figure 5: Percentage of lessons in sub-sample at each content level

Lesson more coherent

Coherence was defined by the panel as "the (implicit and explicit) interrelation of all mathematical components of the lesson" (Hiebert *et al*, 2003: 196). As can be seen from Figure 6 below, 90% of the Hong Kong lessons were judged to be thematically coherent, with the remaining 10% moderately thematically coherent. This compares very favorably with the other countries in the Study. For example, in the Czech Republic and the United States, only 30% of the lessons were judged to be thematically coherent.

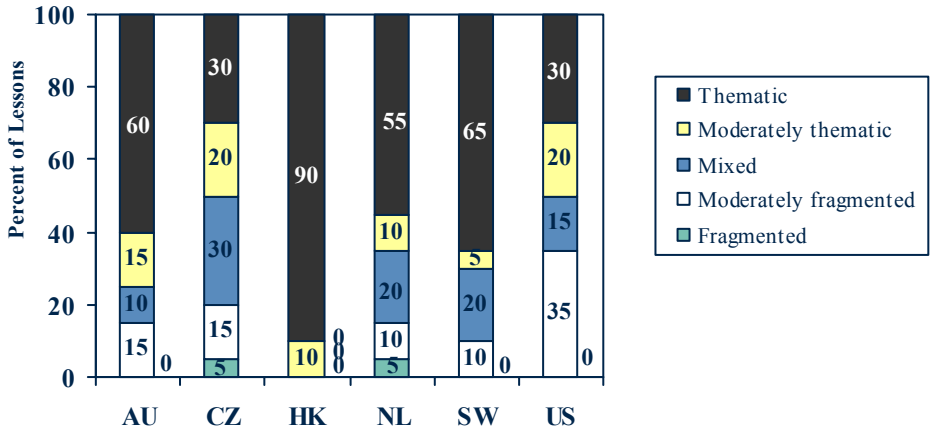


Figure 6: Percentage of Lessons in Sub-sample Rated at Each Level of Coherence

More fully developed presentation

Not only were the Hong Kong lessons judged to be more coherent, their presentation was also found to be more fully developed. Presentation was defined by the panel as "the extent to which the lesson included some development of the mathematical concepts or procedures" (Hiebert *et al*, 2003: 197). Development required that mathematical reasons or justifications were given for the mathematical results presented or used. Presentation ratings took into account the quality of mathematical arguments: higher ratings meant that sound mathematical reasons were provided by the teacher (or students) for concepts and procedures. Mathematical errors made by the teacher reduced the ratings. The results of the judgment of the panel are show in Figure 7 below. It can be seen from Figure 7 that 20% of the Hong Kong lessons were judged to be "fully developed". This percentage is highest among all the other countries, and is in striking contrast with Australia where none of the lessons were classified as "fully developed". If we take into consideration the category "substantially developed" as well, we can see that three quarters of the lessons in Hong Kong were classified as either 'fully developed' or 'substantially developed'. This figure is three times higher than that for the lessons in the Netherlands.

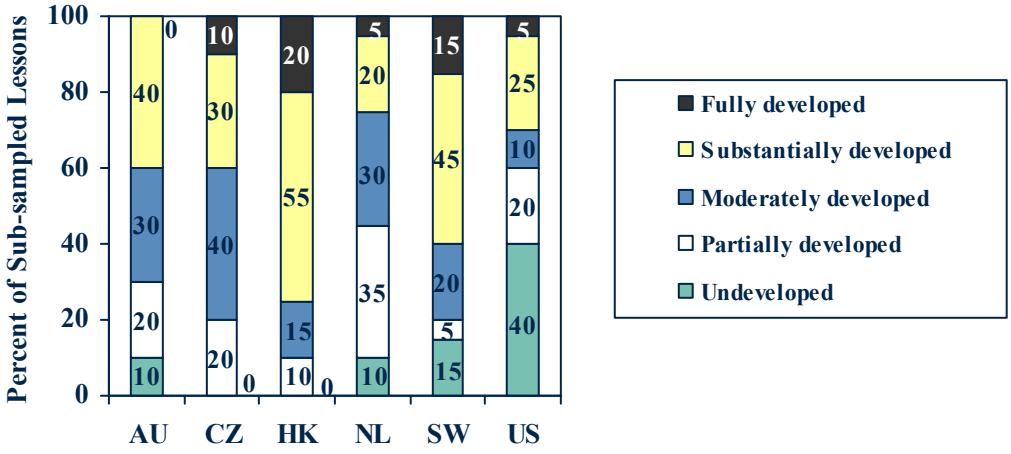


Figure 7: Percentage of Lessons in Sub-sample Rated at Each Level of Presentation

Students more likely to be engaged

As noted previously, the panel did not watch the videotapes (since the exercise was conducted "country-blind") and so could not easily judge whether students were engaged in the lessons or not. From the detailed descriptions of the lessons compiled by the international coding team, the panel made a judgement as to how likely it was that students would be engaged in the lessons. Student engagement

was defined by the panel as "the likelihood that students would be actively engaged in meaningful mathematics during the lesson" (Hiebert *et al*, 2003: 198). A rating of 'very unlikely' (1) indicated a lesson in which students were asked to work on few of the problems in the lesson and those problems did not appear to stimulate reflection on mathematical concepts or procedures; a rating of 'very likely' (5) indicated a lesson in which students were expected to work actively on, and make progress solving, problems that appeared to raise interesting mathematical questions for them and then to discuss their solutions with the class.

As can be seen from Figure 8 below, the panel inferred from the lesson descriptions that students in Hong Kong classrooms were more likely than those elsewhere to be engaged in the lesson. The panel estimated that students in 35% of the Hong Kong lessons were likely to be engaged, whereas in none of the Australian lessons were students likely to be engaged.

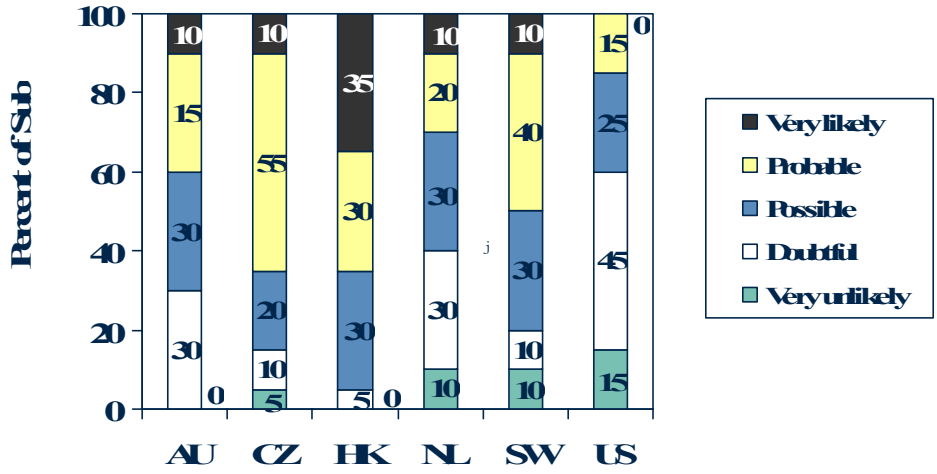


Figure 8: Percentage of Lessons in Sub-sample Rated at Each Level of Student Engagement

Overall quality

Finally, the panel made a judgement on the overall quality of the lessons in terms of "the opportunities that the lesson provided for students to construct important mathematical understandings" (Hiebert *et al*, 2003: 199). Figure 9 below indicates that 30% of the Hong Kong lessons were judged to be of high quality, whereas only 5% of the lessons in Australia and the Netherlands were judged to be so. And in the U.S., none of the lessons were judged to be of high quality. There were also more lessons in Hong Kong than in other countries for which the panel judged the overall quality to be 'high' or 'moderately high'.

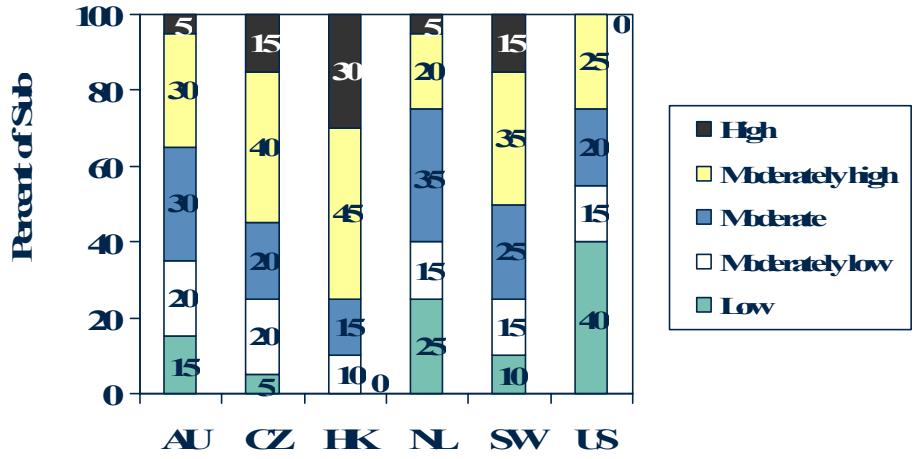


Figure 9: Percentage of Lessons in Sub-sample Rated at Each Level of Overall Quality

Summary

From the results presented above, we can see that the quality of instructional practice in the Hong Kong mathematics classroom was judged by the Mathematics Quality Analysis Group as very high. The mathematics content covered was judged to be relatively more advanced, the lessons were more coherently structured, and the presentation was more fully developed. Given these positive elements of the classrooms, students were expected to be more engaged in the teaching and learning process, and the overall quality of the lessons was judged to be high by the panel.

This presents a picture of instructional practices which is much more positive than that portrayed by the quantitative analysis of the codes.

Discussion

A. Two different pictures?

From the discussions above, it can be seen that the picture of instructional practices in the Hong Kong classroom as portrayed by the judgement of the Mathematics Quality Analysis Group is in stark contrast to the picture as portrayed by the quantitative analysis of the codes. How do we reconcile the apparent inconsistency between the instructional practices as reflected by the two different analyses of the same data set?

It should be noted that in the first picture, instructional practices were portrayed through objectively coding and summarizing activities that happened in the

classroom, whereas in the second picture, the quality of content was judged by the Mathematics Quality Analysis Group based on their expertise and experience. In the international report of the Study, readers are alerted to the small sample size involved in the qualitative analysis and are urged to be cautious in the interpretation of the results. Readers are warned that the sub-sample "might not be representative of the entire sample or of eighth-grade mathematics lessons in each country" (Hiebert *et al*, 2003: 190). Such warning needs to be heeded, for it pertains to the reliability of the analysis results. That is, from a psychometric point of view, the results of the qualitative analysis are deemed to be not very reliable. In addition to the small sample size involved (which is typical of qualitative studies), the very fact that the analysis relied on the judgement of a group of experts means that the results may be "rater-dependent". Given another group of experts with different experience and inclinations, rather different conclusions about the teaching in the Hong Kong classroom may be arrived at, even when the same set of criteria and definitions are followed. In contrast, for the quantitative analysis, since the coding (e.g. number of words spoken by teachers and students) is relatively objective, it is expected that given adequate training, any coder should arrive at more or less the same results.

Since the results of the qualitative analysis are not very reliable statistically, should we discard them and resort only to the reliable quantitative analysis? The quantitative analysis of the TIMSS 1999 Video data, as with all low-inferenced quantitative analysis, has its own limitations as well. Take the number of words spoken by teachers and students in the classrooms as example again. The quantitative analysis of the data computed accurately the number of words spoken by teachers and students in each country, and both the absolute number of words spoken and the ratio between teacher and student words provide relevant information on the kind of interaction that took place in the classrooms concerned. However, every teacher or educator knows too well that the quality of what the teachers and students say in class is far more important that how much they say. But to determine what the teachers and students say are significant or not, the data analysis requires a lot of judgement based on profound experience on the part of the researcher. And in a quantitative study where the emphasis is on low-inferenced data analysis, this is not possible. Hence quantitative analysis may yield results that are highly reliable but not necessarily very meaningful.

Thus, it seems that there is an inherent trade-off between reliability and validity in the analysis of video data. In order to get highly reliable data, we have to restrain from making inferences, and hence we lose out in validity. In order to increase the validity of the analysis of video data, we need to make subjective judegment, with the result that high reliability is difficult to attain. This is rather like Heisenberg's

Principle of Uncertainty in physics[11]. It seems that if we want to get highly reliable and objective information, we have to lose out in the meaningfulness or validity of the data. On the other hand, since qualitative analysis involves the judgement of "experts" based on their experience and expertise, different groups of experts may yield different results. So the information we obtain cannot be very reliable.

Which, then, is the "real" picture of mathematics teaching in Hong Kong? Is the unreliable expert judgment of the Mathematics Quality Analysis Group "real"? Or does the quantitative analysis of the data of the Study fail to reveal the subtlety of the complexity of classroom teaching?

The answer depends on whether you prefer a very reliable description of the activities that happened in the classroom, or whether you can tolerate some lack of reliability and want to learn more about the experts' view on the quality of teaching and learning in the classroom. The crux of the matter is: in determining the quality of teaching, should we rely on objective summary of data, or should we rely on subjective judgment of experts? Perhaps a synthesis of the two gives a picture nearer to the reality.

B. The Traditional East Asian Culture and the High Achievement of East Asian Students

Given that the results of the quantitative analysis of the Hong Kong data in the Study (which shows that instructional practices in Hong Kong are not very conducive to quality learning) are at least part of the "real" picture in Hong Kong, how can we explain the high achievements of students in Hong Kong and other East Asian countries in international studies of mathematics achievement? Also, do the findings of the Study throw any light on the negative attitudes of East Asian students towards mathematics?

First, the traditional teaching in Hong Kong as revealed by the quantitative analysis of the Study may be explained by the underlying cultural values in East Asia. In a replication of Ma's study (Ma, 1999) in Hong Kong and Korea (Leung and Park, 2002), it was found that although the teachers in the study were in general competent in mathematics, they often deliberately taught in a procedural manner for pedagogical reasons and for the sake of efficiency. They seemed to believe that it would be inefficient or even confusing for school children to be exposed to rich concepts instead of clear and simple procedures. This illustrates

[11] The Principle states that the more precisely the position of an object is determined, the less precisely its momentum is known in this instant (see Cassidy, 1992).

very well the pragmatic philosophy in the East Asian culture (Ko, 2001; Shusterman, 2004).

Secondly, the underlying cultural values shared by the East Asian students may also explain both their high achievement and negative attitudes towards mathematics. In the East Asian culture, there is a strong stress on the virtue of humility or modesty. As the author pointed out elsewhere:

> Children from these countries are taught from when they are young that one should not be boastful. This may inhibit students from rating themselves too highly on the question of whether they think they do well in mathematics, and so the scores may represent less than what students are really thinking about themselves. On the other hand, one's confidence and self image are something that is reinforced by one's learned values, and if students are constantly taught to rate themselves low, they may internalize the idea to result in really low confidence. Furthermore, the competitive examinations systems coupled with the high expectations for student achievement in these countries have left a large number of students classified as failures in their system, and these repeated experiences of a sense of failure may have further reinforced this lack of confidence.(Leung, 2002: 106)

Given that East Asian students possess such negative attitudes towards mathematics and hold such low self-concept in mathematics, why do they perform so well in international studies of mathematics achievement? Paradoxically, from the standpoint of the East Asian culture, one may argue that this negative correlation between students' confidence in mathematics and their achievement is something to be expected:

> Over-confidence may lower students' incentive to learn further and cause them to put very little effort into their studying, and hence result in low achievement. This is exactly the kind of justification for the stress on humility or modesty in the East Asian culture. The Chinese saying "contentedness leads to loss, humility leads to gain" illustrates the point well. (Leung, 2002: 106)

In addition, the stress in the East Asian culture on diligence and practice may have also contributed to the high achievement of their students (Park and Leung, 2003). Underlying this stress on diligence and practice is the traditional East Asian value that attributes success more to effort than to innate ability (Leung, 2001). The ultimate root of the stress on diligence and practice is the underlying Confucian cultural values which emphasize strongly on the importance of education and a high expectation for students to achieve. Under the influence of this philosophy, learning or studying is considered a serious endeavour, and students are expected

to put in hard work and perseverance in their study. This is reinforced by a long and strong tradition of publication examination, which acts as a further source of motivation for learning. This high expectation on students to achieve provides an important source of motivation for students to learn well and to excel.

C. Implications

Given the methodological complexity of interpreting video data, and the cultural explanation of the high achievement of East Asian students, what lessons can educators from East Asia and elsewhere learn from the results of the Study?

Implications for East Asian countries

First, just examining the results of the quantitative analysis of the Hong Kong data in the Study may prompt us to call for radical changes in instructional practice in the Hong Kong classrooms, and by inference the classrooms in East Asia as well. However results from the qualitative analysis of the data present a different picture. Some readers may tend to embrace the qualitative results since they are more consistent with results of the achievement data (and for readers from East Asia, the results from the qualitative analysis of course look more pleasing!), and dismiss the quantitative results as invalid. But it should be stressed that the quantitative analysis is done using a relatively more objective (at least more objective than the qualitative part of the analysis) methodology and utilizing a larger and more representative data set (compared with the qualitative analysis). So the findings should not be dismissed lightly. A more balanced view of the two sets of results is that they represent two aspects of the same reality. They complement each other in giving a picture closer to the reality of the Hong Kong (and East Asian) classroom.

Seen in this light, findings of the qualitative analysis of the video data should remind educators in East Asia of their strengths in terms of instructional practices in mathematics. In particular, the expectation that students should learn a relatively advanced level of content with an appropriate degree of abstraction ought to be retained. Simply reducing the difficulty of the content in order to make mathematics more accessible to the general student population is an endless retreat. At the same time, teachers from East Asia should treasure their tradition of teaching mathematics in a coherent manner. They should also continue their effort to fully develop their lessons so as to keep their students engaged in the mathematics. Teachers should of course try to make the lesson lively and enjoyable for students through introducing various activities in their classrooms, but the goal should be to induce students to be interested and engaged in the subject matter of mathematics itself rather than in the lively activities per se. These clearly have implications for reforms in curriculum content, teaching methods, the kind of teachers to be recruited, and the kind of teacher education needed for teachers to perform their job.

On the other hand, it should be admitted that dominance of teacher talk may not be the best kind of activities for effective mathematics learning. Also, despite their students' success in international studies of mathematics achievement, educators in East Asia need to ask themselves whether the fact that the majority of the problems student solve are unrelated to real-life is in itself consistent with their ideal of a good mathematics education. The challenge for mathematics educators in East Asia is to promote more student participation in meaningful learning without compromising their strengths in instructional practices as identified above.

Implications for other countries

If instructional practices and the resulting student achievement are so much related to the underlying culture, what are the implications for countries outside East Asia?

First, students' mathematics achievement in international studies should be viewed in conjunction with their attitudes towards mathematics and mathematics learning. Although negative attitudes of students may not necessarily disadvantage their achievements, the negative attitudes themselves should be considered part of the attainment of the curriculum in the countries concerned, and educators should be alarmed by such negative attitudes. Curriculum documents in countries around the world always include enjoyment of study as part of the aims of education, irrespective of the culture of the countries. So a high student achievement in international studies should not relegate efforts to promote students' interest in their study. We don't want students to do well in mathematics while hating it.

Secondly, simple transplant of classroom practices from high achieving countries to low achieving ones would not work. Since teachers and their teaching are so much influenced by the underlying cultural value, one cannot transplant the practice without regard to the cultural differences. Culture by definition evolves slowly and stably with the passage of long periods of time, and there is simply no quick transformation of culture. What we can learn from another culture through comparative studies is to identify not only the superficial differences in educational practice, but the intricate relationship between educational practice and the underlying culture. Through studying these relationships in different cultures, we may then begin to understand the interaction between educational practices and culture, and through identifying the commonality and differences of both the educational practices and the underlying cultures, we may then determine how much can or cannot be borrowed from another culture.

Conclusion

For many sectors of the community, especially the media, the attention of international comparative studies is usually focused on the relative position of countries in the league tables generated from the studies. For other people, especially the educational policy makers, such international studies sometimes provide an impetus or excuse for educational changes. But very often, such changes are made without a careful consideration of the complex context in different countries within which the achievement and classroom instructions under study are situated.

However, the primary purpose of these international studies is not for countries to compete with each other. Nor should the results of comparative classroom studies be used rashly to justify the classroom practices of the high achieving countries. The significance of such international studies should lie in the rich data set they generate, serving as mirrors for educators to better understand their system. And any changes in educational policies should take into account the rich data set as well as the different cultural values that generate such richness.

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INNOVATION OF MATHEMATICS TEACHING WITH ICT

- The case of dynamic geometry software: Geometric Constructor -

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Geometric Constructor is one of the dynamic geometry softwares used in Japan. There are three versions (DOS, Windows, Java), some web applications using GC/Java. One of the features of Geometric Constructor is the existence of powerful users. By the collaboration with teachers, this software has been able to be developed. One of the core of collaboration is lesson study. Three lessons are described in this paper. The whole-class discussion is suitable to the lesson with Geometric Constructor.

INTRODUCTION

Geometric Constructor (*GC* : in short) is one of the dynamic geometry software (*DGS* : in short) like *cabri*, *Geometer's Sketch Pad* etc., which is developed by the author. Since 1990, it has been used in many schools (mainly junior high schools) in Japan. And we had made lesson study with many teachers. In this paper, I would like to overview the outline of the innovation with *GC*, focusing the features of software and examples of lessons.

FEATURES OF GEOMETRIC CONSTRUCTOR

Dynamic geometry software used in Japan

GC is one of the dynamic geometry software. We can construct a geometrical figure. Under dragging some points, we can find some invariants or functionality in the figure. The first version of *GC* was MS-DOS version (*GC/DOS,* 1989 -). According to the development of computer and network, we have developed the windows version (*GC/Win*, 1997 -) and make the web site *Forum of Geometric Constructor* to provide software , manuals and other educational resources for practices. Since 2000, I and *Zeta* corporation have developed the Java version (*GC/Java*. 2000 -) to make many content according to the junior high school textbook.

From this project, we make two another approaches. I make some web application (*GC_BBS*, *PukiWiki with GC*) using *GC/Java*, by which we can construct figures and save it at everywhere and every time, without installing specific software. *Zeta* is developing *GCL*(which is one of the dynamic geometry software developed based on macromedia Flash) and *dbook* to make commercial contents according to the textbooks, which will be available in this April. These approaches make a distinct feature of GC from commercial softwares like *cabri* and *Geometer's SketchPad*.

GC has developed as a tool of mathematical investigation for us and as a tool of mathematics teaching for teachers and as a tool for content providers.

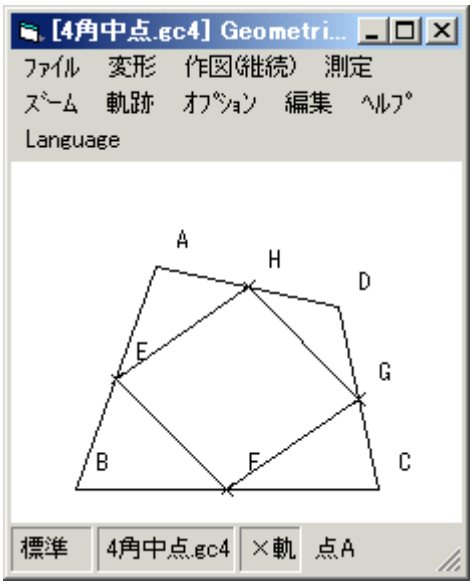


Figure 1: *GC/Win*

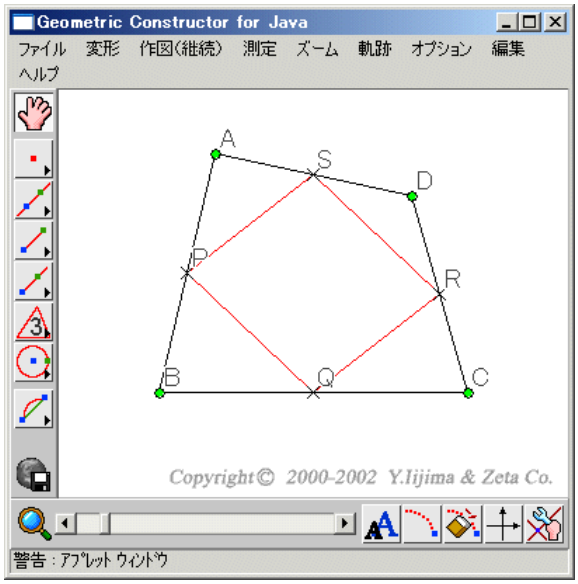


Figure 2: *GC/Java*

Interactive theorem finding through variation of geometrical figure

Schumann (1991) mentioned about interactive theorem finding through variation of geometrical configuration with *cabri*. We can do it similarly with *GC*. We can use a mouse or keyboard to drag. The most typical example of *GC* is shown in Figure 1,2. We can drag points A, B, C, D to deform ABCD and investigate the relation of ABCD and EFGH.

We can make various special cases and make many findings from comparison of them.

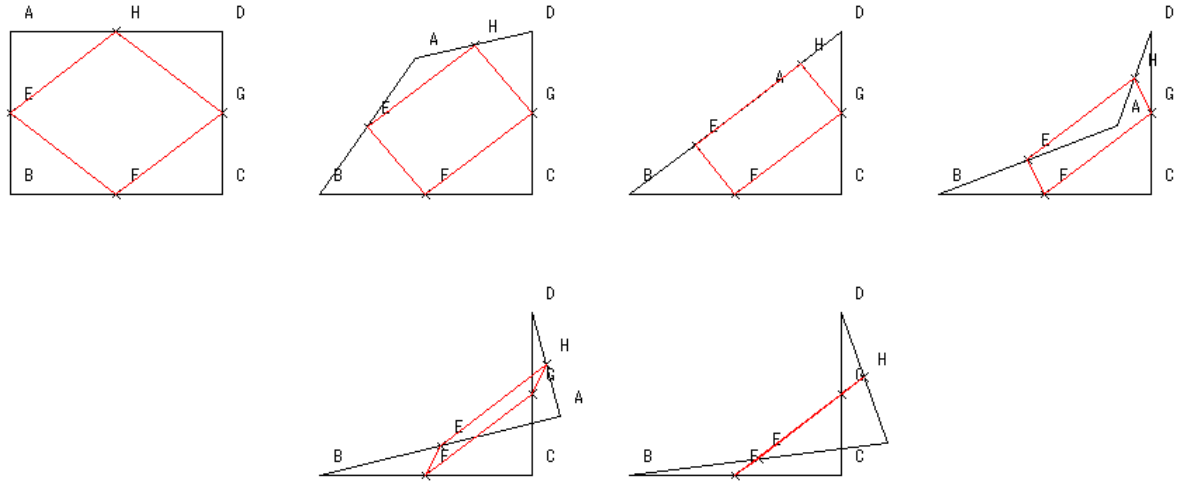


Figure 3:continuous variation by dragging point A

In Figure 4, red line is a bisector half line of angle BAC. By dragging point A, we get Figure 5, by which only few students can find an invariant.

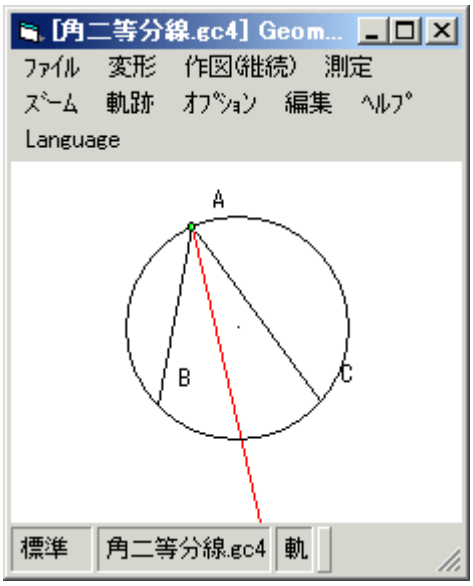


Figure 4: a bisector half line of angle ABC

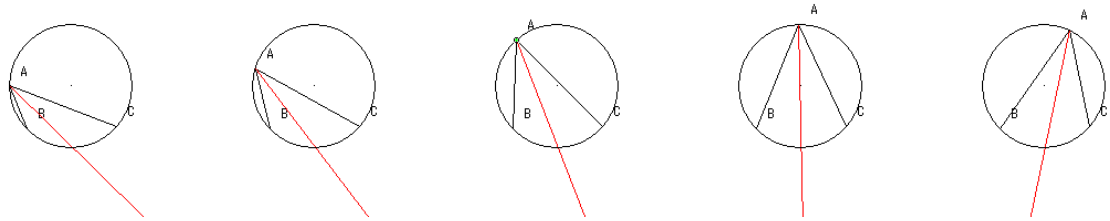


Figure 5:variation of bisector

But, if we make the trace of half line as Figure 6, many students find the fact that line and circle is crossed at the same point.

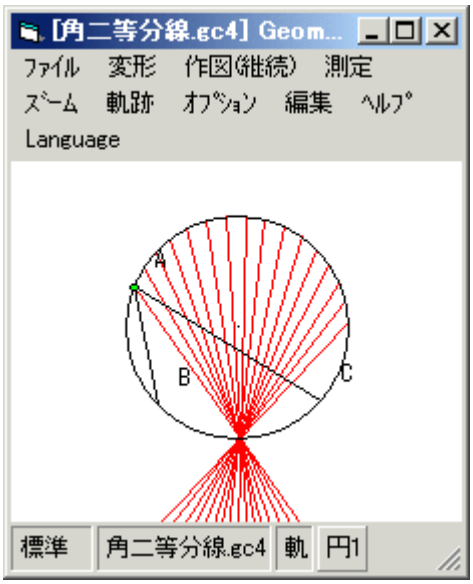


Figure 6:trace of the bisector

We have many examples like these.

Three styles of *GC*: Standalone software, static web contents and web application

We can use *GC* in three different styles, which are connected seamlessly.

According to e-Japan strategy, we will use a computer with projector in an ordinary classroom. We have not enough time to use computers in mathematics teaching (In fact, we have only three hours for mathematics per week in junior high school). So, the basic use of ICT in mathematics is presentation of contents in the CD/DVD or in the Internet. We can use many web contents using *GC* from them. In this case, we use *GC/Java* as a viewer of the static web contents. We can make a presentations, discussions and investigations in mathematics teaching, but we cannot save it (it means static).

If we want to make students experience mathematical investigation, we need to write report and to save figure(it means dynamic). To do so, we have two solutions. One of them is the use of *GC/Win* or *GC/DOS* as a standalone software. Another is the use of *GC_BBS* or *PukiWiki with GC* as web applications.

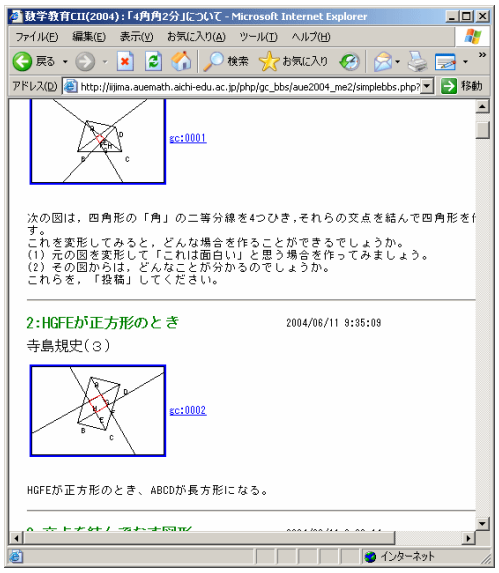


Figure 7:*GC_BBS*

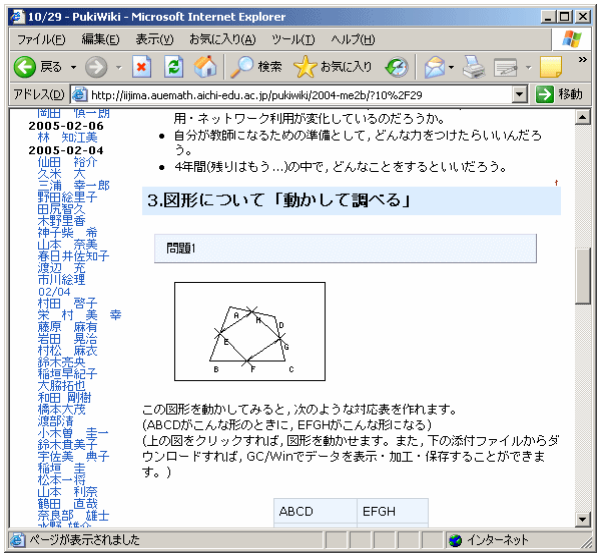


Figure 8: *PukiWiki with GC*

Three modes of *GC/Java*

GC/Java can be used as a viewer for beginners. For beginners, simple is best. But occasionally, we hope to investigate with same contents. In this case, we want to use *GC/Java* as a tool for investigation as same as *GC/Win*. For the sake of this purpose, *GC/Java* has three modes, which can be changed with clicking icon. This is a different feature of *GC/Java* from *cabrijava* and *JavaSketchPad*, which can be used as a viewer only.

Viewer mode : We can use only some functions ; drag, locus, zoom, etc.

Applet mode : We can use some icon for construction.

Window mode : *GC/Java* has the own window. We can use full menu of *GC/Java* and change the size of the window.

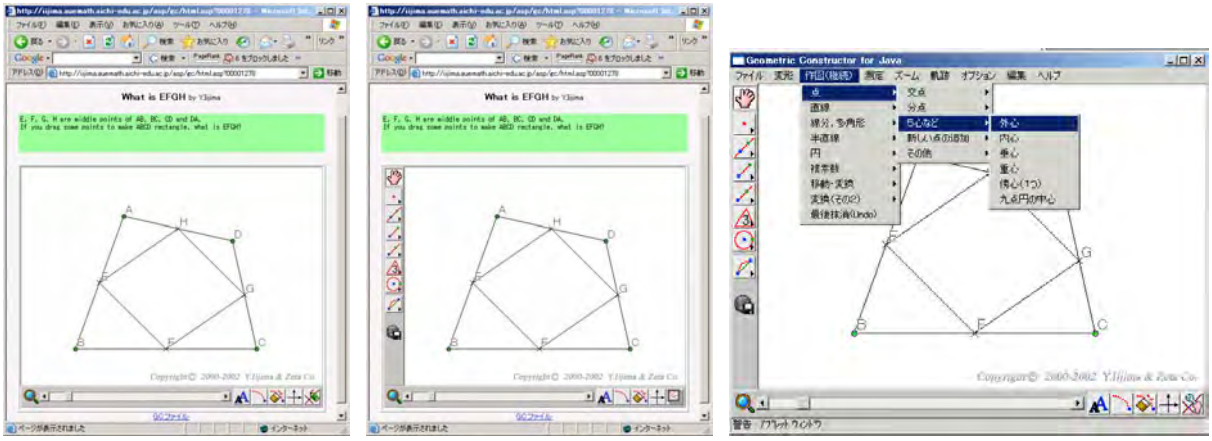


Figure 9: Three modes of *GC/Java*

Web contents with *GC*

We have developed many web contents which can be used in mathematics teaching. The portal site of *GC* is *Forum of Geometric Constructor*. There are some kinds of samples about mathematics topics, questionings, records of mathematical investigations etc.

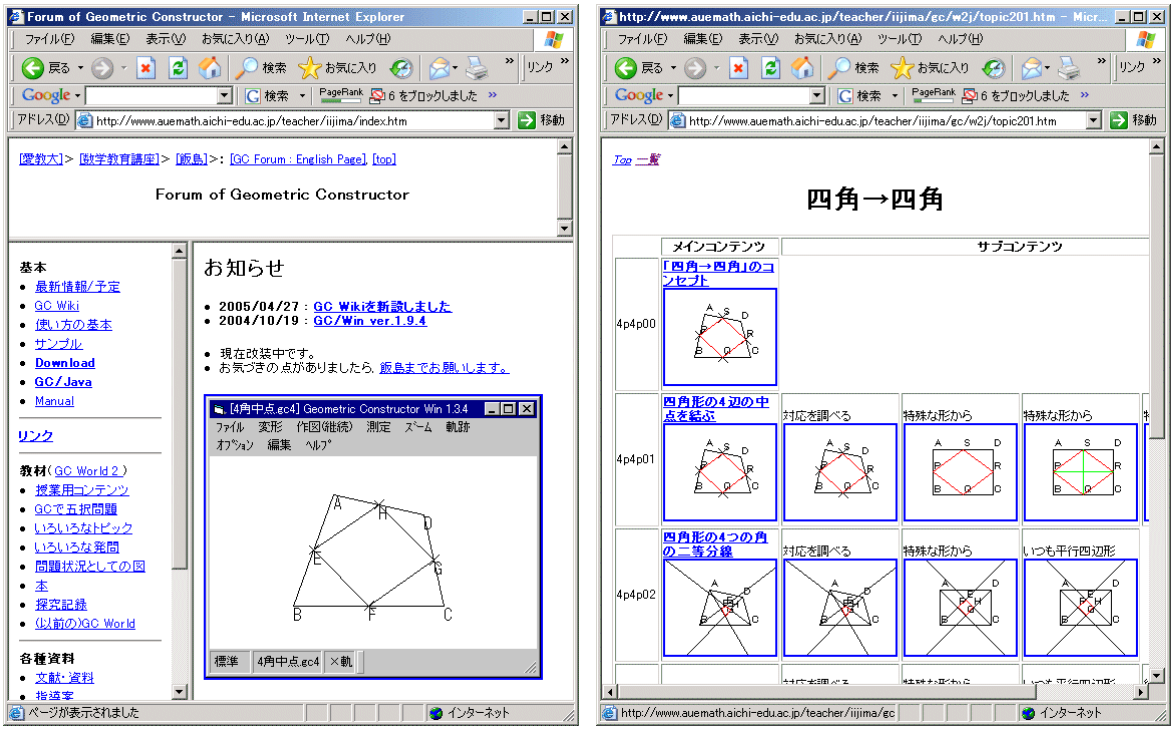


Figure 10: Forum of Geometric Constructor

Uehara provides many web contents suitable for mathematics teaching in junior high school in the following site.

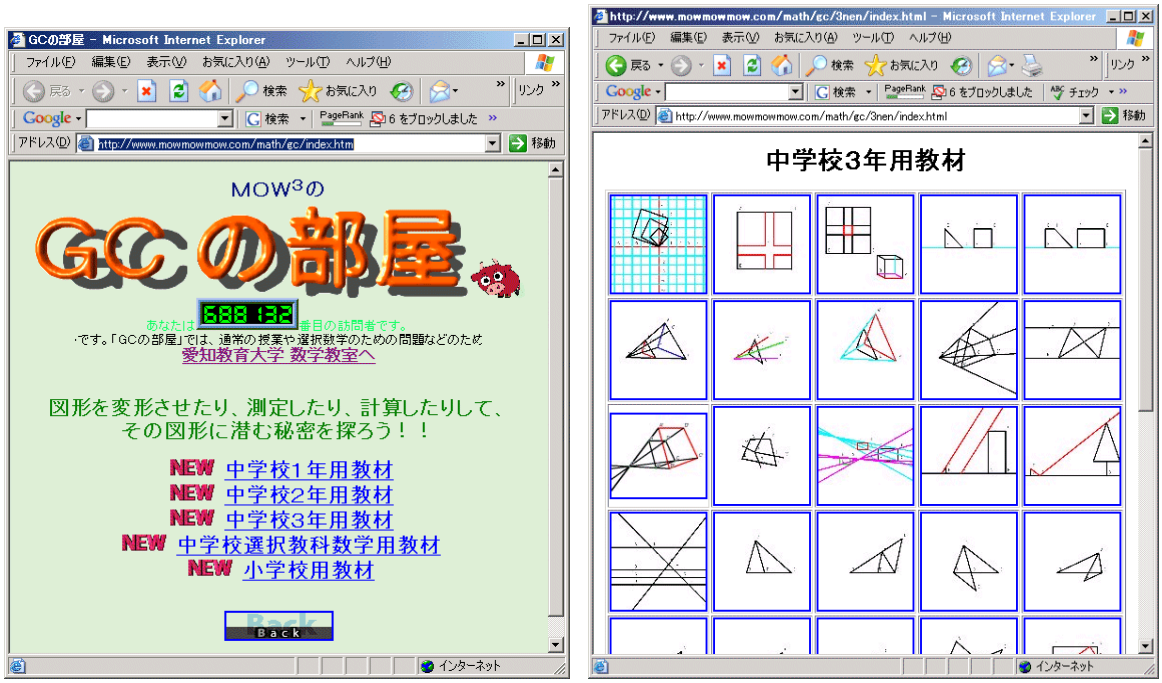
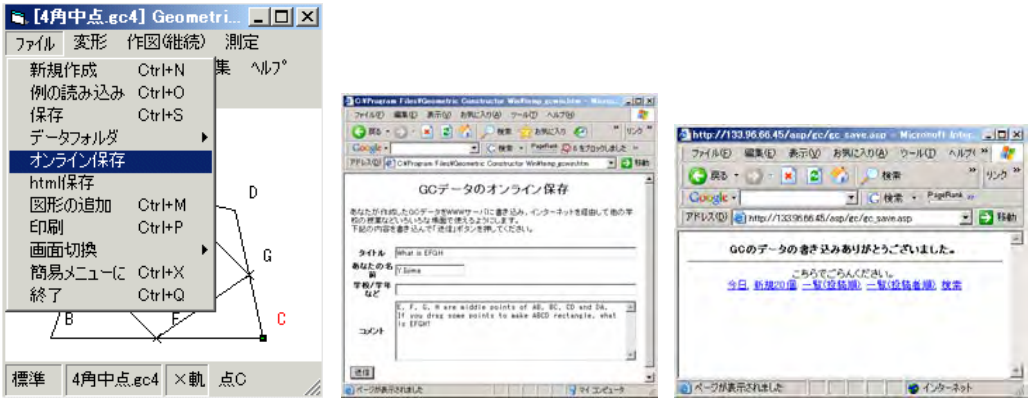


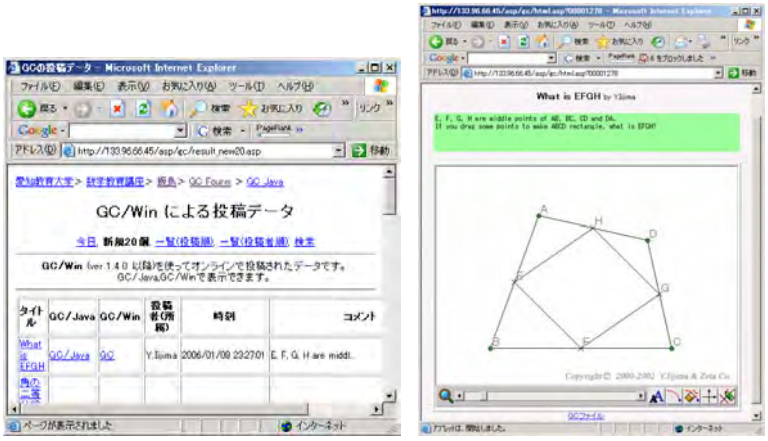
Figure 11: Mow[3]'s Room of *GC*

Instant web content making with *GC*

It is easy to make a web content using *GC/Java*. To make a web content we can use the on-line saving function of *GC/Win*; (1) we make a figure, (2) make a title and a question and (3) save it on the server (iijima.aeumath.aichi-edu.ac.jp). We can use them immediately and globally.



select online saving making question saved



list of contents instant web content

Figure 12: instant web content with *GC/Java* made by *GC/Win*

COMMUNITY OF POWERFUL USERS OF GC

One of the features of *GC* is the existence of the community or powerful users (mainly, teachers in junior high schools), which has made many discussions about software, web contents and lesson study.

For discussion, we use our mailing list, from which we get about 1,000 e-mails for year. They have made many requests about new functions of *GC*. If I implement such a new function of software or a new prototype of content, I upload them on the server, and propose to discuss in the list. In a week, we have some cycles of check and re-making. In this way, we have developed many web contents to enjoy and discuss in this community.

This collaboration between researchers and teachers is very satisfactory for us. And it will be more important in the future. I think that we cannot realize it without ICT.

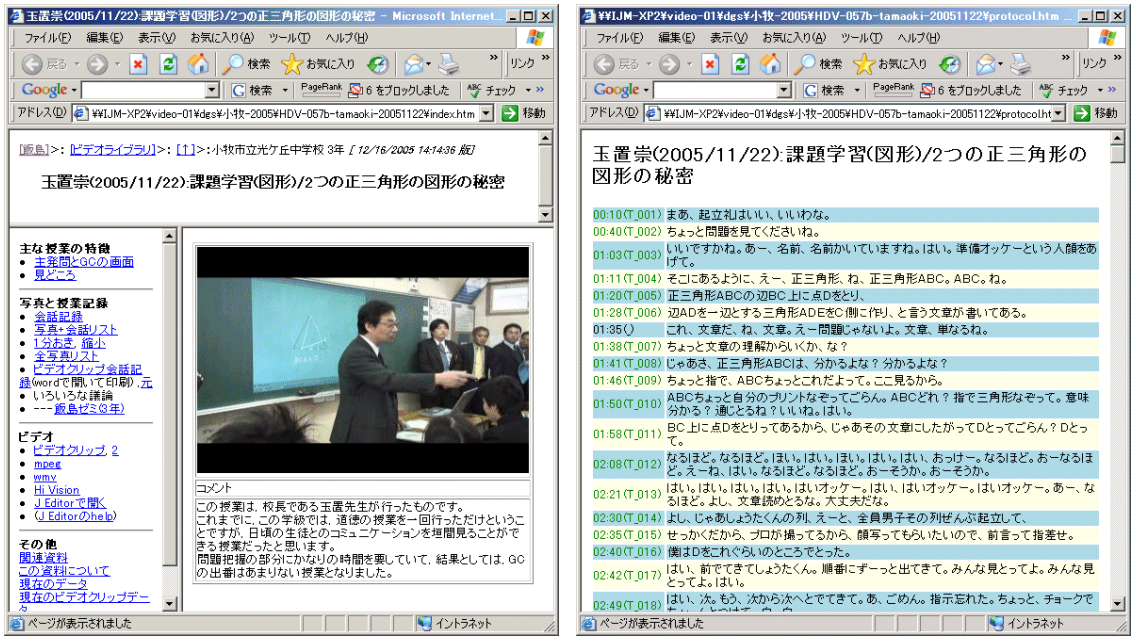
LESSON STUDY AND DIGITAL VIDEO LIBRARY

One of the cores of discussion in this community is lesson study, which is the theme of this conference. In many case, about 10 teachers attend to observe and discuss the lesson. But, many other teachers cannot attend, because of their own job at own schools. So we send them copies of video tape (or a video file in CD/DVD, Figure 13) and discuss about the lesson in the mailing list.

Figure 13: DVD contained video files and resources of the lesson

Since 2002, we archive them to a digital video library. It contains full-video files, lesson plans, transcripts and video-clips of the lesson (if possible). Members of our community who has ID can access the library at their schools and can discuss about lessons archived in it. And if possible, we use it in our undergraduate and in-service teacher training. If our university and schools will be connected with broader network, the importance of such video library of lessons will be increased.

Now, most comprehensive library about video clips of lessons with IT is http://www.nicer.go.jp/itnavi/, which contains 430 examples (about all subjects). In which, three lessons with *GC/Java* by our members are contained.



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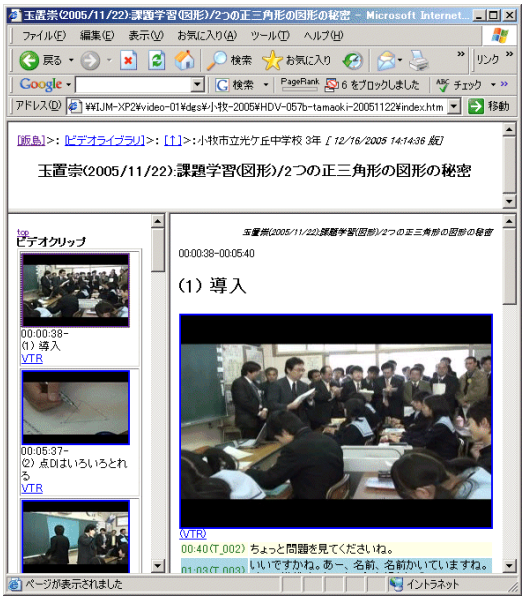


a list of the clips(left) and the first video clip and transcript(right)

Figure 14: lesson archived in the video library

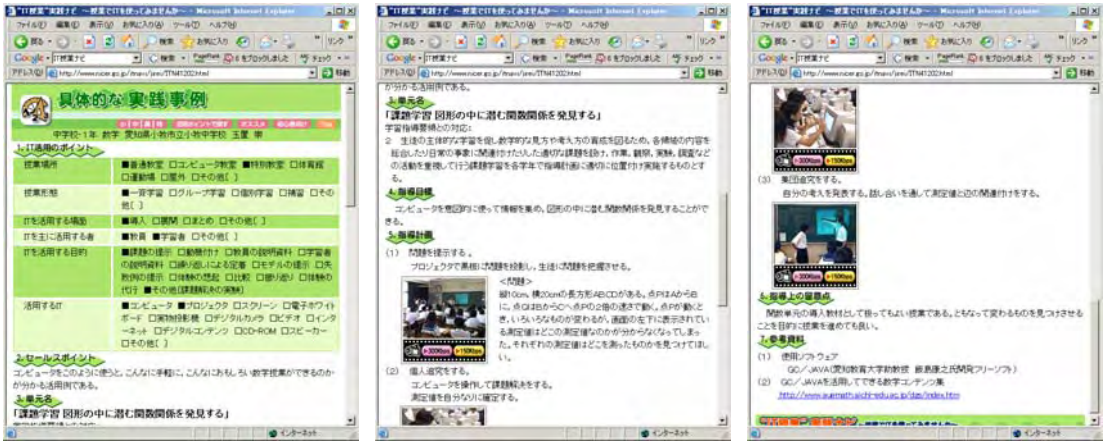


Figure 15: lesson archived in library of NICER

INTERNAL STANDARDS TO USE GC IN MATHEMATICS TEACHING

If we can make softwares and contents to be used easier in schools, it is not easy to make a good practice for many teachers, who has no experience of lesson with ICT. We have to share the key concepts or standards appropriate to lesson with ICT.

Hershkowitz(2002) shows the standards of CompuMath team as follows;

1. Inquiry (observing, hypothesizing, generalizing, and checking) is a desirable mathematical activity.
2. Mathematical activity should be driven by the goals of understanding and convincing.
3. Proving is not only the central tool for providing evidence that a statement is true but should also support understanding why it is true.
4. Mathematical activity should take place in situations that are meaningful for the students.
5. Mathematical activity must stem from previous knowledge (including intuitive knowledge).
6. Mathematical activity should be largely reflective.
7. Mathematical language (notion systems) fosters the consolidation of mathematical knowledge; it should be introduced to students when they feel the need for it.
8. Technical manipulation is not a goal in itself but a means to do mathematics.
9. Computer tools support and foster the above and beyond.

These standards are important for us, and we have some internal standards to have a lesson with *GC*, which may be implicit or explicit. I will sketch them as following section.

Use ordinary know-how for the whole-class discussion effectively.

In Japan, many teachers emphasize the whole-class discussion, which is effective in the lesson with *GC*. It is important to emphasize ordinary know-how for the whole-class discussion to make a good practice and to make relax teachers and students.

For example, in many case, we project *GC/Java* on a blackboard as Figure 16, not on a screen. In a whole-class discussion, *Bansho* is important. Teachers want to write several mark and keyword and whole proof in some case. They can write them easily in the case of blackboard.

Figure 16: *GC/Java* projected on a blackboard

Do not use technology excessively.

We don't spend much time to manipulate *GC*. We spend about 5 minutes in a presentation with *GC* in usual case. We spend about 10-20 minutes for individual/group investigations in the case of 90 minutes lesson. We spend more time to understand, formulate, hypothesize, and discussion. More excessively, less mathematically, we think.

We use open approach in many case.

With *GC/Java*, we can pose problematic situation without words. Inevitably, we use open questions. There are a variety of formulations of the situation. The process of formulation from the situation is important in the lesson with *GC*.

Elegant use of *GC* does not always make a good practice.

We want to make a good problematic situation, not nice presentation of the computer. In many case, with elegant use of *GC*, students feel no problem. According to the problem, more primitive and unskilled use of *GC* is better to make a good practice,

Not only objectively, but also affectively, subjectively.

Our problem solving should be objectively. But we emphasize students' affection and involvement to the problem. From one situation, we can make a variety of problems, a variety of processes. According to the students' findings, ideas, suppositions, utterances, awareness, teacher have to navigate a nice process of problem solving.

SOME LESSONS WITH GC

Angle of circumference: construction and group investigation with *GC*(2001)

In this lesson, we could use many but old computers. So teacher used *GC/Win* and students used *GC/DOS*. Teacher talked how to construct and measure the angle of circumference.

Problem : investigate about the angle ACB.

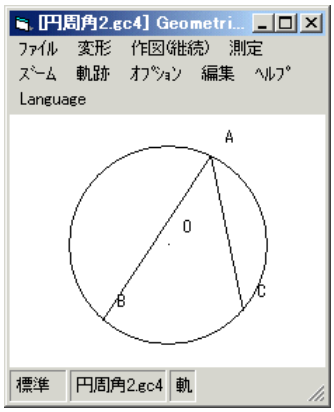


Figure 17: angle of the circumference

Figure 18: problem posing and first investigation

Students found many things. Some students dragged points A and B, and found that if the center O is on the segment AB(which means AB is a diameter of circle), the angle BAC is 90 degrees. Some students found that if they drag point A, then angle BAC is constant, but if they drag point B and C, then angle is changed. Following conversation was interesting;

1	S1:	(She is dragging point A) The size of this angle does not change. It is not interesting.
2	S2:	Yes, it is not interesting.
3	S3:	Teacher, this does not change. It is not interesting.
4	T1:	What does not change?
5	S4:	This angle.

6 T2: Why you feel not interesting?

7 S5: I hope the size of this angle to change.

8 S6: Why does not the size of this angle change?

They investigated some other special cases, and they were involved in the situation.

Some minutes later, teacher projected a student's display to the blackboard with the use of video camera and projector (Figure 19).

1 T1: Drag the point (A).

2 S1: (He is dragging the point A)

3 T2: What do you find from this?

4 S2: The size of this angle does not change.

5 T3: (To the classroom) Do you find same thing?

6 T4: Other group found other thing. Talk about it, dragging it.

7 S4: If point A pass the segment BC, the size of this angle changes.

8 T5: Yes. Let's observe this on your computer, and find what happens.

Figure 19 projection to the blackboard and second investigation

Following is the conversation of same group who talked that this is not interesting.

1 S1: This angle is 54 degrees.

2 S2: Yes, it is just.

3 S3: Very comprehensible! (The sum of two angles is 180 degrees.)

After these investigations (it took 19 minutes), teacher asked to make the formulation of the theorem and the proof of it (33 minutes remained).

Relation of Angles : presentation and group investigation with paper(2004)

In this lesson, teacher use a computer with projector. Teacher showed the relation of the angles which was learned at previous lesson. And he dragged the vertex and show some other relations.

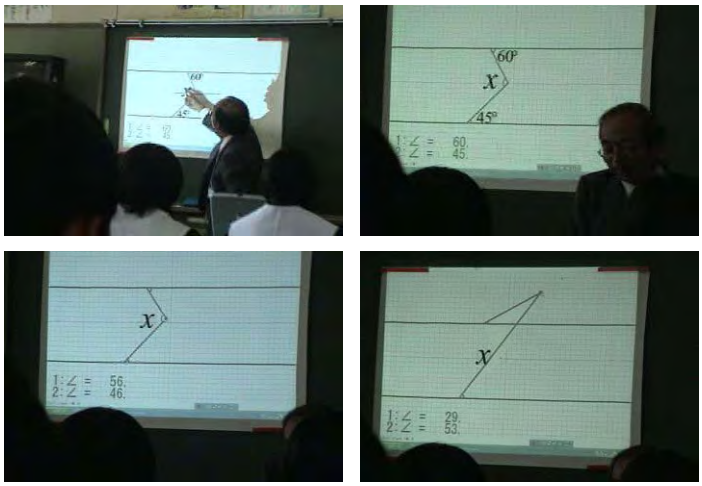


Figure 20: Presentation of some relations by dragging

And he asked to make new problems by the change of some condition of original figure. A student said that he wanted to make more vertices and angles. Teacher made requested figure, but it was not interesting. So, teacher asked other problem. Other student said what if, lines are not parallel. And teacher showed it. And he showed today's main problem: What is the relation of these angles in this figure?

(Teacher spent 5 minutes in this presentation.)

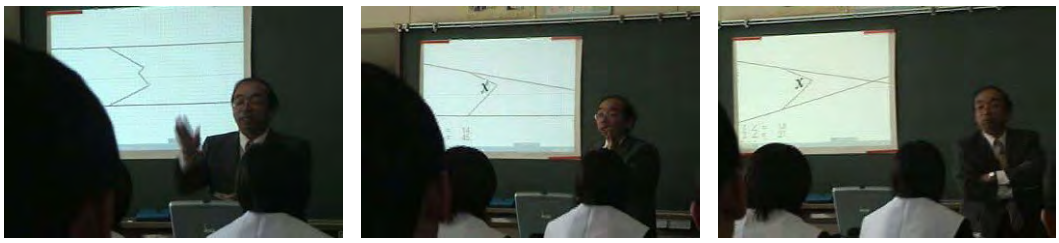


Figure 21: teacher changes figure according to student's request

Students started their investigation with worksheet individually. And next, they had group discussion. Teacher walked around into groups, and sometimes he gave a advice (Figure 22).

Figure 22: individual, group investigation and teacher's advice

Each group summed up each idea on the board, and explained it on the blackboard, and they discussed the comparison and relation of ideas.

Lastly, teacher used *GC/Java* to show extension in two minutes.

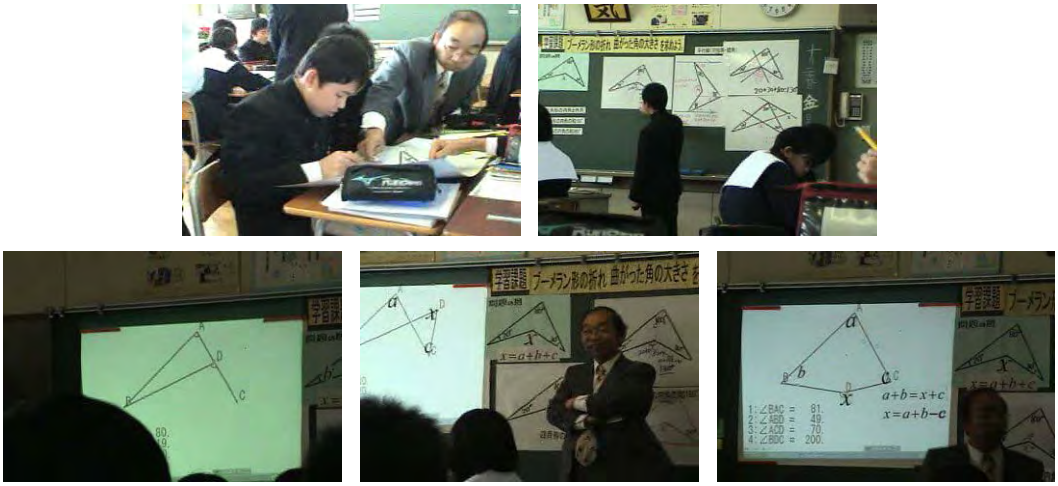


Figure 23: Whole class discussion

Quadrilateral composed by four angle bisectors of a Quadrilateral: group investigation and whole class discussion (1992)

First lesson was done in the computer room. Teacher asked to remember the problem about quadrilateral composed by four middle points of four segments of a quadrilateral, which was learned last year with other software. He talked that if we change the shape of ABCD, then the shape of EFGH change. He manipulated *GC* to present students how to investigate and how to manipulate *GC*. He changed the shape of ABCD to rectangle, rhombus, square, parallelogram, trapezoid and so on. He checked the relation (showed in table 1) only verbally.

Figure 24: presentation of how to investigate with problem learned previously

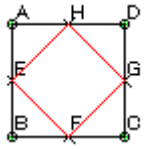
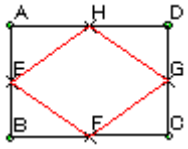
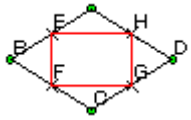
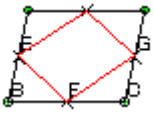
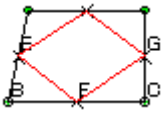
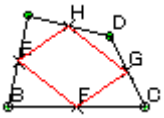
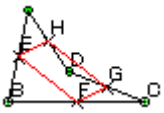
ABCD	EFGH	
Square	Square	
Rectangle	Rhombus	
Rhombus	Rectangle	
Parallelogram	Parallelogram	
Trapezoid	Parallelogram	
Kite	Rectangle	
Quadrilateral	Parallelogram	
Wedge	Parallelogram	

Table 1:relation of quadrilaterals (1)

He showed today's problem.

There is ABCD. We draw angle bisectors of four angles, and name the intersections of bisectors E, F, G, H. We want to investigate this figure like that investigation. We want to investigate the relation of ABCD and EFGH.

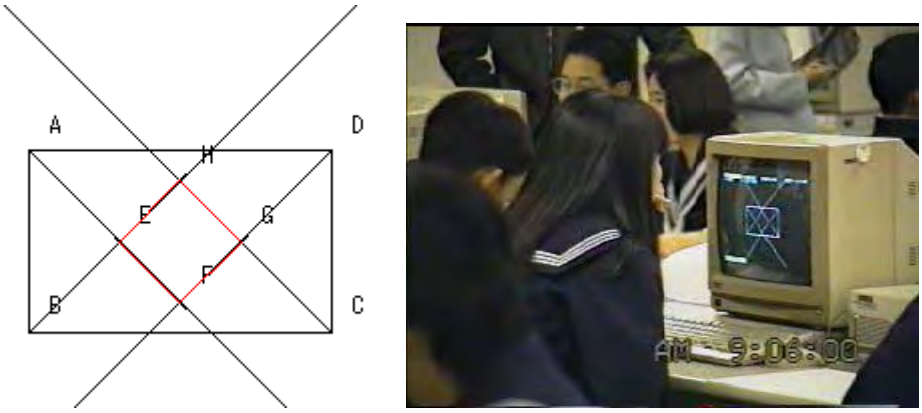


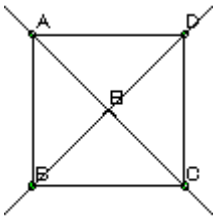
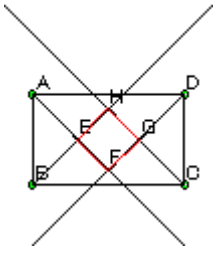
Figure 25: quadrilateral composed by intersections of angle bisectors

He asked what shapes of ABCD do we investigate, and sum up students' answers to the table on the blackboard. And he said. Firstly, suppose the result, and investigate the figure with GC, and write results and sketches on the worksheet.

Figure 26: group investigation with *GC/DOS*

Students investigated in pair for 10 minutes.

And teacher and students made up following table in whole class discussion.

ABCD	EFGH	EFGH (supposition)	Sketch
Square	Point		
Rectangle	Square		

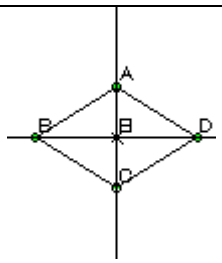
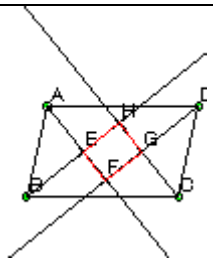
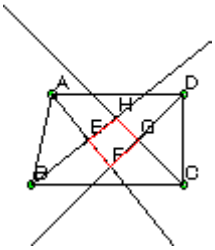
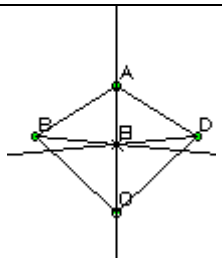
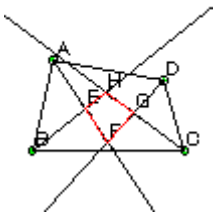
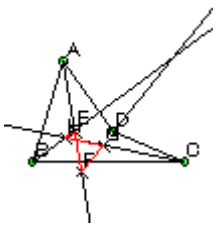
Rhombus	Point		
Parallelogram	Rectangle		
Trapezoid	∠HEF=∠HGF =90°		
Kite	Point		
Quadrilateral	Quadrilateral		
Wedge	Two triangles		

Table 2:relation of quadrilaterals (2)

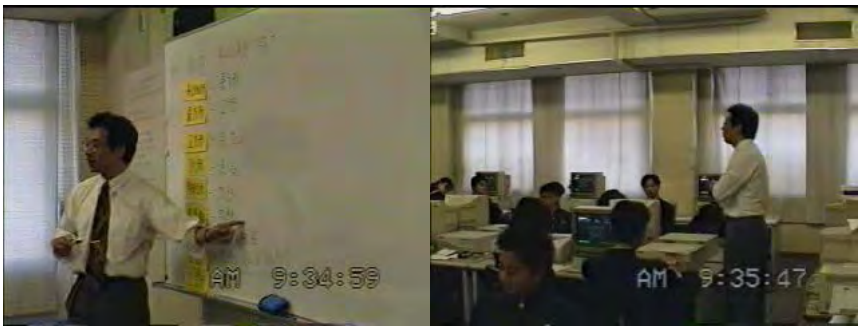


Figure 27:Whole-class discussion

1 T1: What is a difference between your supposition and observation.
2 S1: Parallelogram is …
3 T2: Parallelogram is ?
4 S2: Parallelogram is missed. (Students laughed.)

This student pointed out that parallelogram was supposed in some column but he could not observe it. Other student pointed out rhombus and trapezoid too. They got problems.

- Is this table valid?
- Can we make EFGH parallelogram, rhombus, trapezoid?
- If we can not, why?

In next lesson in the ordinary classroom (several days later), to investigate these problems, the teacher picked up these cases, and asked to make proofs.

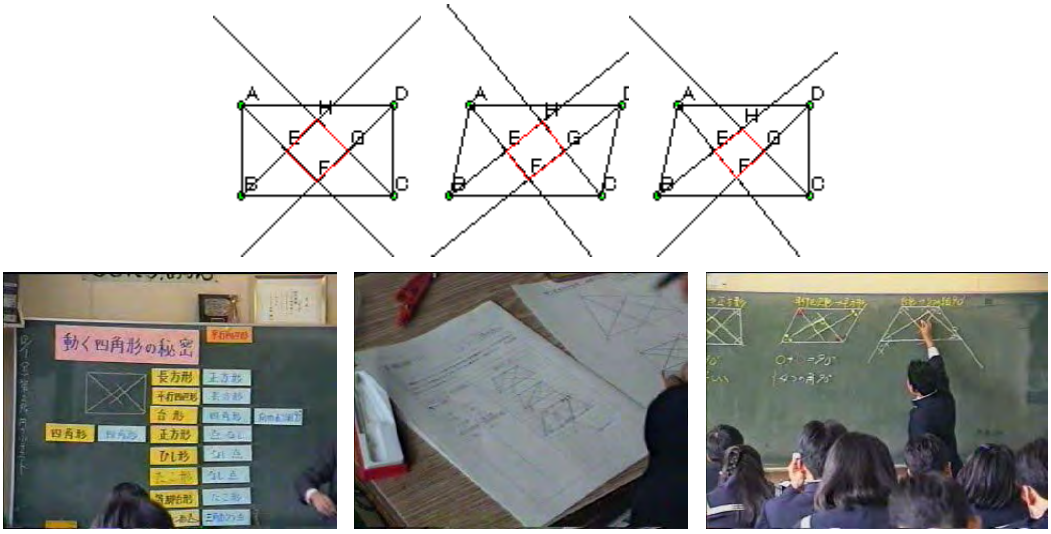


Figure 28:making proofs about the special cases

Secondly, they think about the general case.

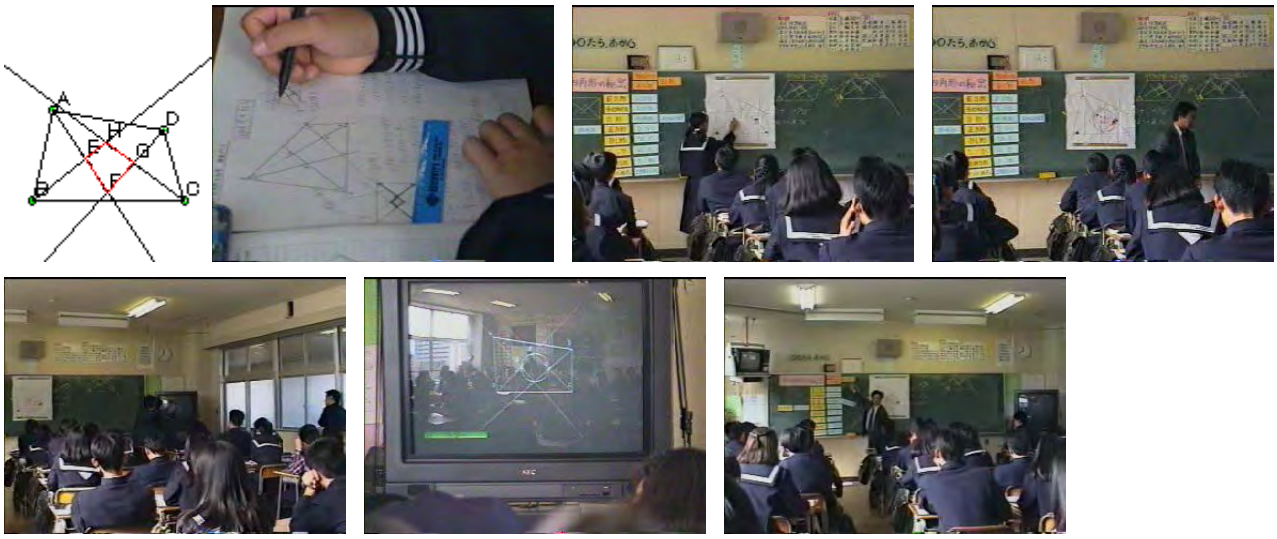


Figure 29: whole-class discussion about the general case

They found EFGH was a cyclic quadrilateral in any case, therefore EFGH can not be parallelogram, rhombus. (Some students pointed out that EFGH could be a trapezoid. This finding was not expected by the teacher.)

Teacher presented the circle always inscribed to EFGH with *GC*. In fact, this was true in the case of wedge. There was relation about two triangles, which was not expected.

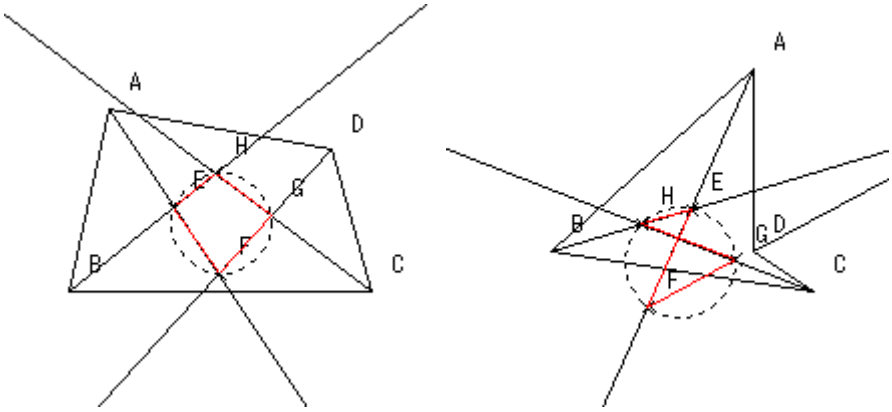


Figure 30: EFGH is always inscribed by a circle

DISCUSSION

Mathematical activity can be supported and fostered with ICT

With computer, we can investigate mathematical problems deeper and wider. We have experienced this since 1990. But, how many teachers have experienced and enjoyed deeper and wider investigations with computer? Not so many, I suppose. This is a serious problem for the expansion of the use of ICT in mathematics teaching in Japan.

Know-how of the whole-class discussion is appropriate with ICT

According to the e-Japan strategy, we will use a computer with a projector in a ordinary room. But, one way presentation is more tedious for students than traditional chalk-and-talk style. To be more attractive, we should use the presentation interactively. Is it difficult? No, I think. Because, many teachers have the know-how about the whole-class discussion in Japan. I hope it make the breakthrough of the Japanese style lesson with ICT.

How to make the *de facto* standards for mathematics teaching with ICT in Japan

In fact, many teachers do not use ICT in mathematics teaching in Japan. I think that one of the reasons is the lack of the standards for mathematics teaching with ICT in Japan. It is not good for us, of course. But, what can we do in this condition?

I think the answer is to make *de facto* standards, which may be possible with ICT as an infrastructure for teachers.

Of course, it is not so easy. But we can provide softwares and contents from our servers. We can make a community to discuss and collaborate.

Lesson study is important to collaborate with teachers in Japan

Many teachers want to make a good practice. Using ICT is not a goal in itself. It is a means to do a good practice. To collaborate with teachers, lesson study is important.

For example, we had a lesson study at Hikarigaoka junior high school at Komaki on 22 November, where we had a comparison between lessons with *GC* and without *GC*. Over 100 persons came to observe and discuss the lessons. They discussed eagerly. We made resources (transcriptions, video-clips, etc.) for the discussion, and had a meeting on 23 December at Komaki. Over 60 persons attended and discussed.

CONCLUSION

The history of *GC* is the history of collaboration with teachers. Designing and implementation of the software is my task. But to device user-friendly interface, the monitoring by teachers is necessary. I have experienced my own mathematical investigations with *GC*, and proposed problems to teachers. They have accepted them and revised them suitable to their students. The core of collaboration has been lesson study. To make a good practice, we have had many discussions before and after the lesson. That produced many mathematical problems, know-how, lesson plans and hints for improvement of software and contents. In this process, ICT has been useful for us. We can provide software and contents with our server in the Internet, and we can discuss almost everyday with mailing list. Now, we are trying to make and use the digital video library of lessons. It is not easy to make a good lesson study, but it is challenging for us.

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Web Sites

Forum of Geometric Constructor : http://www.auemath.aichi-edu.ac.jp/teacher/iijima/

Mow^3's Room of GC : http://www.mowmowmow.com/math/gc/

Navi for lesson with IT http://www.nicer.go.jp/itnavi/

MATHEMATICS LESSONS IN KOREA:
TEACHING WITH SYSTEMATIC VARIATION

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Hongik University (Korea)

To investigate the features of Korean mathematics lessons, the data from the Learners' Perspective Study (LPS) was analyzed. A cursory review of the LPS data gives the impression of a very traditional style of teaching, a salient feature of which is the dominance of teacher talk and reticence of students. Instruction seemed to focus more on the content rather than the process of mathematics, with concepts often stated directly using formal mathematical language.

In a more fine-grained analysis, one lesson judged to be "typical" in each of the three schools was selected for study based on the 'theory of variation.' The results show that there were rich variations in concepts, procedures, and practicing exercise. In particular, a kind of systematic and continuous variations is identified. These variations started with a certain simple basic situation, and one aspect of the situation was then varied at a time until a target form was reached. It is argued that these systematic variations constitute a kind of exploration on the part of the students. These variations were carefully designed by the teacher, leading students to discern a certain set of attributes of the concepts involving the final situation. Coupled with systematic variations in the exercise given to students where they have an opportunity to practice the application of the concepts systematically in class and/or at home, such systematic variations will create the necessary condition for critical attributes of the object of learning to be experienced by the students.

Introduction

In the past decade or so, there has been increasing interest in the study of mathematics classrooms in East Asian countries, or countries falling under the so-called Confucian-heritage culture (CHC), the dominant culture in East Asia. However, relatively little has been published in the international literature on classroom practices in the CHC country of Korea. In this paper, characteristics of the Korean mathematics classroom that are deemed to be conducive to effective learning are identified through an analysis of the Korean data of the Learners' Perspective Study (LPS). Then the classroom characteristics identified are interpreted in terms of the underlying cultural values that they share with other East Asian countries.

Learners' Perspective Study

The LPS, a video study of the mathematics classroom, is characterized by in-depth documentation of the student perspective over several lessons in the same classroom. The methodology of the LPS offers an informative complement to the survey-style approach of the TIMSS video study. A research design of LPS predicated on a nationally representative sampling of individual lessons, as in TIMSS, inevitably reports a statistically-based characterization of the 'typical lesson'.

In the LPS, one teacher from each of three schools in each participating country was sampled for study, and a series of 10 to 15 consecutive lessons taught by the teacher were videotaped. The teachers chosen were judged to be competent teachers in their respective countries. The study combines videotape data with participants' re-constructions of classroom events. Three cameras were employed in the videotaping; a "Teacher Camera," a "Student Camera" and a "Whole Class Camera." An audio-video mixer was used for on-site mixing of the images from the teacher camera and the student camera to provide a split-screen record of both teacher and student actions. The integrated images were used for stimulated recall in interviews conducted immediately after the lessons to get students' reconstructive account of the teaching and learning (Clarke, 2004).

Theory of variation[1]

To identify mathematics classroom features, a learning theory espoused by Marton (1999) is utilized in the analysis of the Korean data. Marton hypothesized that variation, simultaneity, and discernment were critical to learning, and studies by Runesson (1999) and Mok (2000) showed that Marton's theory of variation had a demonstrated potential in revealing the salient characteristics of classroom features that are related to student learning.

The theory of variation was developed from the work of Marton and Booth (1997), which described how an 'enacted space of learning' was constructed through the creation of certain dimensions of variation for the experience of the students. According to Marton *et al* (2003), learning is a process in which learners develop a certain capability or a certain way of seeing or experiencing. In order to see something in a certain way the learner must discern certain features of the object. Experiencing variation is essential for discernment, and is thus significant for learning, and Marton *et al* (2003) argued that it is important to attend to what varies and what is invariant in a learning situation.

[1] Theory of variation is based on Phenomenography, which was developed by a Swedish research group in early 1970s. The word 'phenomenography', coined by Marton in 1979, was derived from the Greek words 'phainemenon' and 'graphein', which mean appearance and description respectively. Thus 'phenomenography' concerns about the description of things as they appear to us. According to phenomenography, a way of experiencing something is defined in terms of the critical aspects of the phenomenon as discerned and focused upon by the experiencer at the same time. Nobody can discern an aspect of a phenomenon without experiencing variation in a dimension which corresponds to that aspect (Marton & Booth, 1997; Pang, 2003). This provides a basis for the theory of variation.

In parallel with Marton's theory of variation, a theory of mathematics teaching and learning, called teaching with variation, has been developed by Gu (1994). Gu's theory was based on a series of longitudinal mathematics teaching experiments in China, and was heavily influenced by theories of cognitive science and constructivism. According to this theory, meaningful learning enables learners to establish a substantial and non-arbitrary connection between their new knowledge and their previous knowledge (Ausubel, 1968). Classroom activities can be developed to help students establish this kind of connection by experiencing certain dimensions of variation. The theory suggests that two types of variation are helpful for meaningful learning. One is called "conceptual variation", and the other is called "procedural variation" (Gu *et al*, 2004).

Conceptual variation consists of two parts. One part is composed of varying the connotation of a concept: standard variation and non-standard variation. The other part consists of highlighting the substantial features of the concept by contrasting with counterexamples or non-examples. The function of this variation is to provide learners with multiple experiences from different perspectives.

Procedural variation is concerned with the process of forming a concept logically and/or chronologically (scaffolding, transformation), arriving at solutions to problems, and forming knowledge structure (relationship among different concepts). The function of procedural variation is to help learners acquire knowledge step by step, develop learners' experience in problem solving progressively, and form well-structured knowledge.

Multi-dimensional Variation and Developmental Variation

While the two kinds of variations suggested by Gu are potentially powerful tools for analyzing classroom events, the terms "conceptual variation" and "procedural variation" may be misleading. The adjectives "conceptual" and "procedural" may remind readers of the terminology of "conceptual understanding" and "procedural understanding" coined by Hiebert (1986), which are used differently from the meaning of "conceptual" and "procedural" as defined by Gu. Gu's terminology may give the impression that "conceptual variation" and "procedural variation" are disjoint, but in fact according to Gu's own definition, procedural variation is also related to the formation of concept. So the terms "conceptual variation" and "procedural variation" do not reflect very well the meaning they are supposed to represent as defined by Gu.

In this paper, the term "multi-dimensional variation" will be used to denote what Gu termed "conceptual variation" because the term refers to enhancing conceptual understanding through multiple representation and varied examples of a given concept. Along with conceptual variation, the term "developmental variation" will be used to substitute for Gu's "procedural variation", since this variation helps the

learners to construct knowledge structures through progressively acquiring the knowledge.

For example, in one of the lessons videotaped, the teacher familiarized students with the concept of linear equations in two unknowns through comparison with linear equations in one unknown. The teacher reminded the class that equations with one unknown and those with two unknowns are similar in the sense that a root should satisfy the equation when substituted into the unknown(s) of the equation. But the two are different because the number of roots is different. This explanation helps students to understand linear equations with one unknown and two unknowns by contrasting the similarities and differences of the two concepts, and is thus considered a "multi-dimensional variation."

Another example of multi-dimensional variation is found in a lesson from another school. There the teacher introduced a new concept (ratio of areas) through concrete examples in everyday life: the fact that the amount of ink needed to print a photo depends on the area of the photo. This connection between an abstract mathematical concept and a concrete example in real life can be interpreted as a multi-dimensional variation as well as "mathematization" in Freudenthal's terms (Freudenthal, 1983).

An example of "developmental variation" is identified in a lesson from the third school videotaped in this study. In the lesson, the teacher provided a variety of situations by presenting a pouch with colored stones and then changing a certain colored stone to another colored stone. Based on this "experiment," students observed that the probability increased from 0 progressively until eventually it reached 1. This was then generalized into the properties of probability. So students acquired the knowledge through experiencing progressive problem solving.

In fact, these notions of variations are similar to the "mathematical variability principle" by Dienes (1973), and the "duality of mathematical concept" suggested by Sfard (1991). According to this theory of variation, the "space of variation" consists of different dimensions of variation in the classroom, and they form the necessary condition for students' learning in relation to certain learning objectives. For the teacher, it is crucial to consider how to create a proper space of variation focusing on critical aspects of the learning object through appropriate activities. For the learner, it is important to experience the space of variation through participating in constituting the space of variation.

For the data analysis in this paper, the patterns of variation critical to learning will be described in two aspects: what the multi-dimensional and developmental variations are and how they are created. Studies by Runesson (1999) and Rovio-Johansson (1999) support the hypothesis made by Marton (2000) that variation is a key for comparing the difference in practices between the East and the West. Marton argued that the most important difference between the Chinese/Japanese classes and those in the U.S. was the difference in the pattern of variation. Chinese and Japanese students learned to approach the same mathematics problem in different ways, whereas the

American students learned to apply the same approach to different but similar problems.

Sample, data collection and analysis

Following the methodology of LPS, three schools in the urban/metropolitan community of Seoul were sampled for study. To preserve anonymity, the three schools are referred to as school H, school K and school W in this chapter. One grade 8 mathematics teacher in each of schools H, K and W judged to be competent by the local professional community was selected. The teacher had at least five years of experience as a qualified teacher. One of the grade 8 classes taught by the teacher was then selected for study, and a continuous sequence of at least 10 lessons were videotaped for the class.

The videotaped lessons were then viewed carefully, and a preliminary analysis was preformed on the data. Then a lesson in each of the three schools judged to be "typical" of lessons in the series was chosen for a more fine-grained analysis. Table 1 shows the background characteristics of the three sampled schools and the sampled teachers, as well as information about the lessons chosen for detailed analysis:

Table 1. Background characteristics of the sampled schools and lessons

	School H	School K	School W
Type of schools	Girls' school	Co-educational	Co-educational
SES of parents	Mostly middle class		
Teacher Gender (age)	Male (47)	Female (32)	Female (33)
Teaching Experience	18 years	6 years	7 years
Class size	36	34	37
Duration of lesson	45 minutes		
Topic of lesson	Linear equations with two unknowns	Area of similar geometric figures	Properties of probability

Results

Preliminary Analysis

A preliminary analysis of all the videotaped lessons shows that the Korean classrooms in this study, like the classrooms in other East Asian countries, were characterized by the dominance of teacher talk and reticence of students when compared with Western countries (Leung and Park, 2005). The number of words spoken by the teachers and the students in all the lessons in the three schools was

counted and their ratio computed, and the results are shown in Figure 1. As can be seen from Figure 1, the ratios of number of words spoken by the teacher to those spoken by the students vary between 18 and 40, with an average of 28. These ratios are higher than those obtained from the TIMSS 1999 Video Study (Hiebert et al, 2003), especially higher than those for the Western countries in the TIMSS 1999 Video Study (Figure 2)

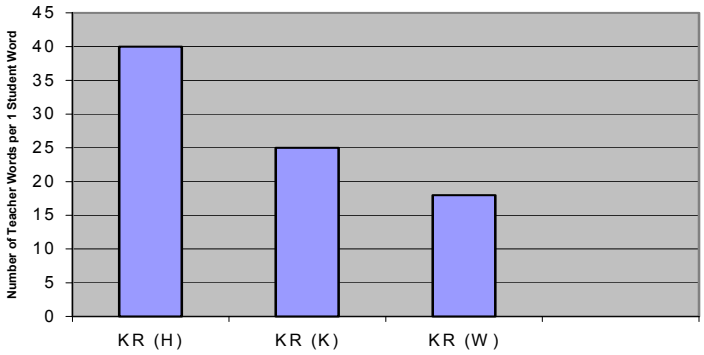


Figure 1. Ratio of number of teacher words to student words in the 3 Korean schools

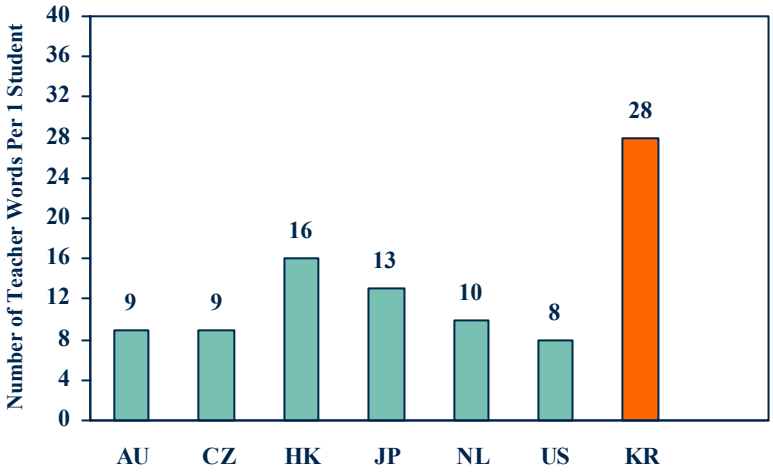


Figure 2. Ratio of number of teacher words to student words in Korean classrooms compared to those in other countries

A cursory review of the video data shows that the teaching in the three Korean schools seemed to focus more on the mathematics content to be learned rather than the process of understanding the content. Mathematics content was delivered efficiently, with mathematics concepts often stated directly. As the teacher in school H remarked during the interview after the lessons:

Teacher of school H:

Of course, there should be lots of student activities. But I found that they distracted the students and made it difficult to proceed with the lesson. Also, the high achieving students seemed to get bored and would sometimes just sit idled. If we have activities in class, those who are not so good don't even know what they are for. Innovative lessons which try new thing in class make everybody tired. Just giving mathematical explanations is much better for both high and low achieving students. People seem to think that inquiry instruction is a good form of teaching that fits the current trend but I do my own explanation and lead the whole class because it (inquiry instruction) tends to loosen the lesson somewhat.

The focus of the lessons seemed to be on the final product rather than the process of arriving at the product. There was much more use of formal mathematical language rather than less formal everyday life language such as metaphors. There was also ample practicing of mathematics exercise during the lessons.

The analysis also shows that the Korean lessons by and large followed a rather similar structure, which we categorize into four stages. In the first stage, which we name *review and induction*, the teacher would usually begin the lesson by reviewing relevant materials covered in previous lessons and prepare the way for the main concepts of the lesson to be introduced. In the second stage, named *exploring new concepts*, the main concepts of the lesson would be introduced and elaborated by teacher-initiated exploration. In the third stage, *examples and exercise*, the main concepts would be illustrated with examples, and students would be directed to work on some relevant exercise. In the final stage, *summary and assignment*, the teacher would summarize the main points of the lessons and assign homework for the lesson.

Fine Grained Analysis

As pointed out above, the preliminary data analysis was followed by a more fine-grained analysis of one lesson in each of the three schools judged to be "typical" of lessons in the school. The further analysis was data-driven, following a grounded theory approach. The lessons were reviewed several times and the variations were identified from the process. Results of this analysis of the three chosen lessons show that during the four stages of the lesson identified above, there were a lot of variations in concepts and practicing exercise. In the discussion below, we denote the variations referred to by two capital letters and a number. The first letter refers to the school (H, K or W) where the lesson took place, the second letter stands for either multi-dimensional variation (M) or developmental variation (D), and the number indicates the order in which the variation occurred in that particular lesson. For example, HD1 means the first developmental variation which occurred in the chosen lesson of school H.

The multi-dimensional and developmental variations identified in the three lessons include:

–Linkage of different concepts, introducing a new topic based on a review of the content covered in previous lessons (HD1 and WD1)

–Consolidation through summary (HM4)

–Learning concepts through comparison and contrast (HM1)

–Linkage between mathematics and concrete examples (KM1)

–Multiple representation of a concept (HM2, KM2)

–Generalization through abstraction (WD2)

Systematic Variation

One particular kind of variation warrants highlighting for discussion. It is a kind of systematic and continuous variation that leads students to understand the concept under discussion. It is interesting to find that such systematic variations were found in each of the three lessons analyzed: HM3 & HM4, KM3 & KM4, and WM1 below can all be classified as this kind of systematic variation.

HM3 & HM4

This is the first lesson of school H. Students were given a series of tasks which were gradual variations to a basic equation, x + y = 5.

	linear equation with 2 unknowns	domain of x and y
basic equation	x + y= 5	natural numbers
	⇩	
task 1	3x + y = 15	natural numbers
	⇩	
task 2	-3x + y = 12	natural numbers
	⇩	
task 3	x + 2y = 6	x values {**-2, -1, 0**, 1, 2, 3, 4, 5, 6, 7}
	⇩	
task 4	x + y = 3	no limitation

In task 1, the domain of the unknowns was natural numbers and only the coefficients were changed from the basic equation. In task 2, the equation included a negative coefficient, and in task 3, the domain of the unknowns was extended to negative integers. In task 4, the domain of the unknowns was further extended to real numbers,

and students were required to draw a graph without going through the process of finding the solution. After solving task 4, the concept that the graph of a linear equation with two unknowns was a straight line in the coordinate plane was well expounded (HM3).

1. T: Let's go ahead and read what's written right below. There is only one line between two points. Therefore, to draw the graph of a linear equation, one would get two roots and graph those two points on the coordinate plane. With those two points, one may easily construct a line.
2. T: So, if we are to draw a line on the coordinate plane we would need to choose two ordered pairs and connect them. What do we get then? We get the graph of a line. There is only one line when we have two points, right? When we connect our points, we get a line.

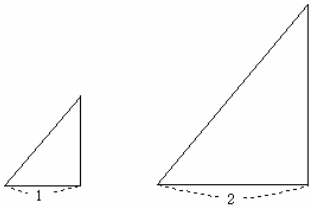
At the end of the lesson, the contents covered thus far were summarized. During this process, the general form of a linear equation with two unknowns ($ax + by + c = 0$) where the domains for the unknowns (x and y) are real numbers was finally introduced. This is a kind of multi-dimensional variation to consolidate and enhance the formation of concepts (HM4).

In the analysis above, we can see that there are systematic variations starting with the basic equation $x + y = 5$ and moving step by step to the general form of $ax + by + c = 0$. In each variation, all but one of the components of the equation concerned are kept constant, so that the effect of the varied component is elucidated.

KM3 & KM4

This is the seventh lesson of school K. Students were given tasks which were variations of a basic diagram. Tasks 1, 2 and 3 were not that different from the basic diagram because students were required to find the ratio of areas when only the type of geometric figure and ratio of similarity were different. Task 4 required students to generalize from what they discovered in the preceding tasks. Here, task 1 serves as scaffolding for tasks 2 and 3 as well as task 4. (KM3)

Task1: Find the ratio of similarity and the ratio of areas of the given figures.



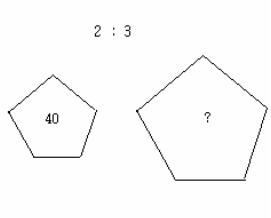
Task2: Find the ratio of areas of the rectangles when the ratio of sides is 1:3.

Task3: Find the ratio of areas of the triangles when the ratio of sides is 2:3.

Task4: Fill in the blanks:

The ratio of areas between similar figures is the ____ of the ratio of sides.
The ratio of areas between similar figures is ____ when the ratio of sides is m:n.

Task5: Compute the area of a large pentagon when the ratio of similarity between two pentagons is 2:3 and the area of the small pentagon is 40.



	geometric figure	ratio of similarity	find
basic diagram	rectangle	1:2	ratio of the areas
	⇩		
task 1	triangle	1:2	ratio of the areas
	⇩		
task 2	rectangle	1:3	ratio of the areas
	⇩		
task 3	triangle	2:3	ratio of the areas
	⇩		
task 4	arbitrary polygon	m:n	ratio of the areas
	⇩		
task 5	pentagon	2:3	the area of one pentagon

The lesson proceeded gradually from the basic diagram through tasks 1 to 4, but task 5, which was given as a review problem of the lesson, made a greater variation to the original problem compared to the preceding tasks (KM4). The geometric figure is not the familiar triangle or rectangle but a pentagon (for which the area is not easily found), and the problem is not to find the ratio of areas but to find the area of one pentagon based on the area of another.

With these systematic variations, students are guided to understand the concept that for a pair of *any* similar polygons, if the ratio of similarity is m : n, then the ratio of

areas is $m^2 : n^2$. Students are expected to be able to find the area of a polygon based on the area of another similar polygon and the ratio of similarity.

WM1

This is the first lesson of school W. The first content to be covered in the lesson was that the probability of an impossible event is 0, that of a certain event is 1, and all probabilities have values between 0 and 1. The teacher did not present the problem in heterogeneous situations but found the probability of a series of situations by continuously changing the color of stones in the same pouch. The teacher drew a pouch on the board, stuck three red magnets in the pouch and showed the students that the probability of selecting a blue stone is 0. Then she replaced one red magnet for a blue one and showed that the probability of choosing a blue stone was then 1/3. By replacing a red magnet by a blue magnet one by one, the class eventually found that the probability of selecting a blue stone when all three stones are blue becomes 1.

In the three examples above, the teaching all started with a certain simple basic equation, diagram or situation. Then only one of the different aspects of the basic equation, diagram or situation was varied at a time, and the variations followed a systematic pattern until the equation, diagram or situation reached a target form. It can be argued that these systematic variations constitute a kind of teacher-initiated exploration or guided ***exploration*** on the part of the students. It seems that the incremental variations were carefully designed by the teacher, leading students to discern attributes of the object of learning or the concepts involving the final situation.

In addition, in all three lessons analyzed above, there were also systematic variations in the exercise given to students. So students after being exposed to systemic variations in the presentation of the concepts would now have an opportunity to practice the application of the concepts systematically in class and/or at home. According to the theory of variation, these combined experiences of the students on the systematic variations of the concepts will help establish their understanding.

Discussion

It was mentioned above that the systematic variations identified constitute a kind of *exploration* on the part of the students. Exploration in the Western context often means students were given open-ended tasks and engaged in free exploratory activities, usually conducted in a small group or individualized setting. This is in contrast to the teacher-directed Korean classroom reported above. However, the fine-grained analysis of the data shows that in the seemingly teacher-directed Korean classroom, students still had the opportunity of exploring mathematics ideas under the close guidance of the teacher. In the words of the variation theorists, such systematic variations will create the necessary condition for different features or critical attributes of the object of learning to be experienced by the students (Marton

and Booth, 1997). In this regard, this kind of exploration is referred to as teacher directed exploration or simply directed exploration in this paper.

The descriptions above fit well with the findings of another study on the classroom practices in Hong Kong and Shanghai. Huang and Leung (2004) reported that the Hong Kong and Shanghai mathematics classrooms in their study were characterized by teacher dominance and student active engagement, with much emphasis on exploration of mathematics and exercises with variation.

The East Asian Culture

How do we account for the classroom practices in Korea as identified in this study? To what extent can these classroom characteristics be attributed to the underlying East Asian culture? In the literature, various scholars have tried to attribute differences in classroom practices and achievements to cultural factors (Watkins and Biggs, 1996; Wong, 1998). In particular, Leung (1999) discussed the traditional Chinese views of mathematics and education which might have an impact on the classroom practices in the current Chinese classroom. Leung (2001) extended the argument from the Chinese classroom to the East Asian classroom and identified features of East Asian mathematics education in contrast to features in the West, and presented the differences in terms of six dichotomies. He argued that the different practices between East Asian classrooms and those in the West are based on different deep-rooted cultural values and paradigms, whether explicit or implicit, that have been built up over centuries. In the next section, we will try to account for some of the classroom practices identified in this study through referring to the underlying cultural values that Korea shares with other East Asian countries.

Teacher dominance and whole class teaching

Teacher dominance and whole-class teaching accord well with the traditional East Asian philosophy which emphasizes integration and harmony (Sun, 1983), in contrast to the Western culture which stresses independence and individualism (Taylor, 1987). Related to this tendency in the East Asian culture, which Yang (1981) labeled as 'social orientation' (as opposed to 'individual orientation'), are characteristics such as compliance, obedience, respect for superiors and filial piety (Lin, 1988, Liu, 1986). East Asians are known to have a tendency of complying with rules or orders more than Westerners, giving rise to a strong tendency for uniformity and conformity (Bond and Hwang, 1986). In such a cultural environment, it is not surprising that classrooms are found to be teacher dominated, with whole-class teaching being commonplace.

Teacher dominance may also be related to the high regard given to teachers in the East Asian culture. In the East Asian culture, the image of the teacher is that of a scholar held with high respect. So it is just natural that in the classroom setting,

teaching and learning activities should be directed by the scholar-teacher. Teacher dominance and whole-class teaching however do not necessarily mean that students are not actively engaged in the lesson. As can be seen from the results of this study presented above, active student engagement is still possible in a classroom where the class size is large and the activities are dominated by the teacher.

Content versus process

It has been reported in the literature that "Chinese teachers held the more rigid view of mathematics being more a product than a process, (and) the more important thing for them in mathematics teaching was to have the mathematics content expounded clearly." (Leung, 1995: 315). The emphasis in the East Asian mathematics classroom was on the mathematics content and the procedures or skills in dealing with the content rather than the process of handling mathematics. There is an underlying belief that

> "the critical attribute of mathematics is its distinctive knowledge structure, and it is this distinctive structure which distinguishes mathematics from other forms of knowledge. So the most important goal of mathematics learning is to understand and get hold of this distinctive knowledge structure, and the foremost task of the mathematics teacher is to help students acquire the mathematics content. The process of doing mathematics is part of the process of learning the content, but the process needs the content as its foundation. Without content, there is nothing for the process to be applied to"
>
> (Leung 2001: 39)

The findings of this study agree well with the reported views above. This stronger stress on the content rather than the process of mathematics also reflects how the nature of mathematics is perceived in the East Asian culture.

The emphasis on directed exploration and practice

The finding in this study on the emphasis of the Korean teaching on directed exploration may seem to contradict the stereotype of the East Asian classroom. The learning styles in East Asia are often portrayed in the literature as "learning by rote" or "passive learning" (Biggs and Watkins, 1996), and the teaching strategies characterized as "procedural" (Zhang, Li and Li, 2003). But results of this study show that behind the seemingly procedural teaching and passive learning, the Korean students are actually heavily involved in exploration when following the prescribed classroom activities designed by the teacher.

On the other hand, the finding that there are a lot of practicing exercises in the Korean lessons is consistent with the stereotype many held for the East Asian

classroom. However, the results of this study also suggest that the exercises that Korean students worked on were not simply repetitive drills, but were carefully designed problems with systematic variations.

In the East Asian culture, practice has always played an important role in the learning process. Actually, the word or term in Chinese for "learning" consists of two characters (学习), and the second character (习) conveys the meaning of practice. So in the CHC tradition, practice is an inherent part of the learning process. The idea of learning without practicing is absurd in the CHC. The well known saying (熟能生巧) which is often translated as "practice makes perfect", reflects this philosophy of learning well. As Confucius put it, "Is it not a pleasure, having learned something, to try it out (i.e., practice) at due intervals?" (*Analects,* I. 1).

Underlying this stress on practice are the traditional East Asian cultural values which lay a strong emphasis on the importance of education and which attribute achievement more to effort than to innate ability. Under the influence of such values, education or study is considered a serious endeavor, and there is a high expectation for students to put in hard work and perseverance in their study and to achieve. This is reinforced by a long and strong tradition of public examination, which acts as a further source of motivation for learning. All these add up to form an important source of motivation for students to learn well and to excel.

Conclusion

As can be seen from what have been presented in this chapter, the analysis of the Korean LPS data utilizing the theory of variation has yielded some interesting results which help reveal the kind of teaching in Korea. Ample practice of mathematics skills does not necessarily imply rote learning or learning without understanding. The analysis in this study shows that there are actually well designed and systematic variations in both the classroom activities and the practicing exercises in the Korean classroom, with the consequence that a lot of exploration is taking place on the part of the students in the teacher-directed classroom. And according to the theory of variation, such experience of variations on the part of the student will lead to understanding. As Leung (2001) pointed out, understanding is "not a yes or no matter, but a continuous process or a continuum." The process of learning often starts with gaining competence in the procedure, and then through "continuous practice with increasing variation" (Marton, 1997), students gradually gain understanding.

Amidst the global tide of educational reform, there is a pressure on governments of East Asian countries, Korea included, to change the educational practices in their countries as well, and a common strategy taken is to send a team of policy makers to a number of "more developed" countries and shop around for new ideas and practices. But too often, those new ideas and practices have not been well tested even in those "developed" countries, and the cultural differences between the East Asian countries and the "developed" countries being visited have not been attended to in the adoption

of the reforms. What is needed in Korea and other East Asian countries for policy decision are systematic collection and analysis of relevant data, and reflection on the strengths and weaknesses of the existing system and the interaction between existing educational practices and the underlying culture. And what is reported in this paper represents exactly one such endeavor.

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Open-ended Approach and Teacher Education

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During 1970s and 1980s, open-ended approach had emerged as a method to reform mathematics teaching of Japanese classrooms and has been spreading around the world. In the 1990s lesson study, a Japanese style of professional development, become known to other countries. During this time educational reform movement in Thailand has been focusing on reforming of student' learning processes and calling for innovative teacher education program. This paper describes how in this era open-ended approach when being integrated with lesson study has become an innovative mathematics teaching to improve teacher education in Thailand.

A BRIEF ON TEACHER EDUCATION

Teacher education or the education of teacher has a long history. In various parts of the world, as Gibb et al. (2003) mentioned, the need for better-qualified teachers has been a critical issue in the minds of parents and educators. In what follows, many classic questions in this field are still debatable issues to the present time: What are the essential characteristics of a professional program for teachers? Should a program for teachers differ from a liberal arts program, and if so, what should be the distinctive features of the treatment of subject matter in each type of program?, and what types of courses in professional education should be required of prospective teachers? (Gibb et al., 2003). This paper emphasizes the last question while concerned with other questions.

Much of the research supports the idea that teacher preparation is important, and that knowledge and skill is built over time, in a coherent program of study. Here are some suggestions from National Council for Accreditation of Teacher Education: High quality educator preparation makes a difference in student learning, teacher preparation increases teacher retention, and teacher preparation helps candidates acquire essential knowledge and skills.

Thailand has undergone many problems in establishing programs for teacher education, particularly in science and mathematics teacher education. Not until the last decade, 36 teachers' colleges and 8 universities of education locating across the country had gained high respectability in providing teachers to elementary and secondary schools. The persons who entered teachers' college and university of education at that time were high-achiever students from various schools. However, after universities of education had been changed to be comprehensive universities thirty years ago and teachers' colleges had been changed to be Rajabaht Institute 10 year ago and now become Rajabaht universities, faculties of education at these

universities have become ′second-class′ faculties in terms of their profile. The graduates feel inferior to graduates from other programs and often have negative attitudes towards their career. This is a crucial problem for most teacher education programs currently.

After the 1999 Educational Acts was enacted, Thailand was put into an educational reform movement. Most school teachers have been attempting to improve their teaching practice. Unfortunately, they lack any innovation to improve their everyday work. Most teachers still use a traditional teaching style focusing on coverage of contents, but they neglect to emphasize students' learning processes and their attitudes toward learning with understanding. More importantly, a number of teachers classify themselves into a reforming group (e.g., master teachers, initiative teachers etc.,) but in effect do not realize that they are still in an old paradigm.

Regarding this point, there are many crucial aspects of the educational reform movement in many countries. Among other things, teacher training is a central issue. Teachers need to learn how to capture students` learning processes and to examine their own practice, etc. However, we lack clarity about how to best design initiatives that involve the examination of practice (Ball, 1996; Lampert, 1999; Shulman, 1992; cf., Fernandez et al., 2003)

Among other alternatives, lesson study is a comprehensive and well-articulated process for examining practice that many Japanese teachers are engaging in (Fernandez, Cannon & Chokshi, 2003). In fact, recently a number of American researchers and educators have suggested that lesson study might be an incredibly beneficial approach to examining practice for US teachers (Lewis&Tsuchida, 1997; Stigler&Hiebert, 1999; Yoshida, 1999; cf., Fernandez et al., 2003).

However, the most difficult part of implementing lesson study in a new classroom culture is that how to get started. Teachers who are new to this approach always insist asking where the first lesson comes from, and how to know that is a study lesson worth for continuing study. To shift from making lesson plan according to the topics ascribed in the curriculum to making lesson plans that will satisfy the long goal as expected in lesson study is not an easy work. It demands changing of teachers' beliefs while challenging them to encounter a new paradigm of teaching mathematics. To solve this problem, using open-ended approach in order to create ***a rich mathematical activity*** is the most important part of making the first study lesson.

A BRIEF ON OPEN-ENDED APPROACH

Open-ended approach originated in Japan during 1970s. Between 1971 and 1976, Japanese researchers carried out a series of developmental research projects on methods of evaluating higher-order-thinking skills in mathematics education using open-ended problems as a theme (Becker and Shigeru, 1997). This approach started with having students engaging in open-ended problems which are formulated to have multiple correct answers "incomplete" or "open-ended". In terms of teaching method,

one “open-ended” problem is posed to the students first, then, proceeds by using many correct answer to the given problem to provide experience in finding something new during the problem-solving process. Mathematical activities generated by open-ended problems are very rich and subtle so as teachers can evaluate student’s higher-order-thinking skills. In a sense, open-ended problem is a good start for creating the first study lesson for the purpose of study in lesson study approach.

Constructing a good open-ended problem is not an easy task. Suggestions for constructing appeared on *The Open-ended Approach: A New Proposal for Teaching Mathematics* may help doing that. However, for the new comers those suggestions are still very difficult.

Japanese teachers have long experiences in developing story problems. Thus, they can implement the suggestions just mentioned in order to make their own open-ended problems. However, for Thai teachers they are familiar with introducing new contents to students through some examples and exercises. It is very difficult for them to organize many mathematical concepts into a problem situation, which is an important part of open-ended problems. This kind of problem situation has to be formulated so that mathematical activity can be naturally generated from it. In what follow, the project introduces the concept of presenting the problem situation in terms of some 3-5 short instructions instead of presenting in terms of story.

In this way, it is easy for students to start mathematical activity from the given open-ended problems. It is also so suitable for teachers to investigate how their formulated open-ended problems have been engaged in by the students. This will be helpful for them to revise their open-ended problems which included in lesson plan. This will be a good start of lesson study.

AN EXEMPLAR OF NTEGRATING OPEN-ENDED APPROACH AND LESSON STUDY

To illustrate how to implement the idea mentioned above into teaching practice, the author conducted a small project with 15 student teachers enrolled in the practicum teaching course. This attempt is an expansion of the meanings of professional development and also the notion of lesson study. Similar to professional development of school teachers, student teachers need to examine their own practice. In the 2002 academic year, the Faculty of Education at Khon Kaen University, in an attempt to improve teacher education program, conducted a project to investigate how student teachers develop their worldview on teaching practice and to investigate how school students in the classrooms using the Open-Approach method of teaching recognize their learning experiences.

Overview

The project was conducted in the 2002 academic year in 7 schools in Khon Kaen province in the northeastern part of Thailand. It is aimed at investigating changes in student teachers' worldview on their professional development when using the Open-Approach method of teaching (Nohda, 2000). The project is also aimed at clarifying how school students recognize their learning experiences. Fifteen student teachers voluntarily participated in this project and 1200 junior highschool students responded to the survey. Those student teachers enrolled in the forth year of the bachelor degree program at the Faculty of Education, Khon Kaen university. According to the requirement of the program, they had to conduct their practicum teaching at their selected schools for one semester (about four months and a half). They had to follow some regular activities designed by the program and had to follow some additional required activities designed by the research project. In what follows, regular activities and required activities for this project are described.

1) The Research Project Settings

1.1 Regular activities requiring all student teachers to do

All student teachers had to teach in Khon Kaen urban area 6-8 periods (about 50 minutes for one period) a week. The school teachers who serve as school supervisors can assign appropriate work to the student teachers. For one semester, the student teachers were supervised 4 times by school supervisors and another one time by supervisors from the faculty. They also had to conduct a classroom research under advisorship of his/her research advisor. Furthermore, they had to attend seminar or to meet their research advisors on every Friday afternoon (approximately three hours).

1.2 Required activities for the research project

Fifteen student teachers who participated in the research project had attended a one-month workshop for constructing lesson plans to be used later in the first semester of 2002 academic year. They were grouped according to school levels they intended to teach. Six were in the 7th-grade group. Five were in the 8th-grade group and four in the 9th-grade group. Coached by the researcher, they spent about 6 hours a day constructing lesson plans using open-ended problems. Ten units of lesson plans to be used for 10 weeks were completed before they went to schools. The remaining 5 units were conducted afterward.

In order to have a chance to share their experiences of teaching by the open-approach method, the 15 student teachers attended a special seminar organized by the researcher on every Friday. In this seminar they expressed their common concerns, interesting points, changes of some particular students' behaviors, and etc. Furthermore, they were expected to develop some ideas in order to conduct classroom research.

During the whole semester, they also had to make a journal related to their experiences of teaching with the open-approach method. This journal was used for discussion in the special seminar on Friday.

1.3 Parts of the Research Results

In response to the aims of the research project, research results will be described in two categories: Change in student teachers' worldview on teaching practice and learning experiences of students in the classrooms using the open-approach method of teaching.

1.3.1 Change in student teachers' worldview on teaching practice

During the first half of the semester all student teachers in the project experienced the difficulty in adjusting to their roles in classroom organization. Participating in Friday seminar made most of the student teachers gradually change their worldviews on teachers' role. The most critical point of change depended upon encountering different experiences of their friends. Sharing experiences with their friends in Friday seminar not only resolved their common concerns but also developed their worldview on teaching practice which in turn reflecting their worldview on professional development. The most important aspect of student teachers' worldview is that teaching mathematics does not mean only focusing on the coverage of content. Emphasizing on students' learning processes, original ideas and also attitudes towards learning mathematics satisfying one's competence is more importantly.

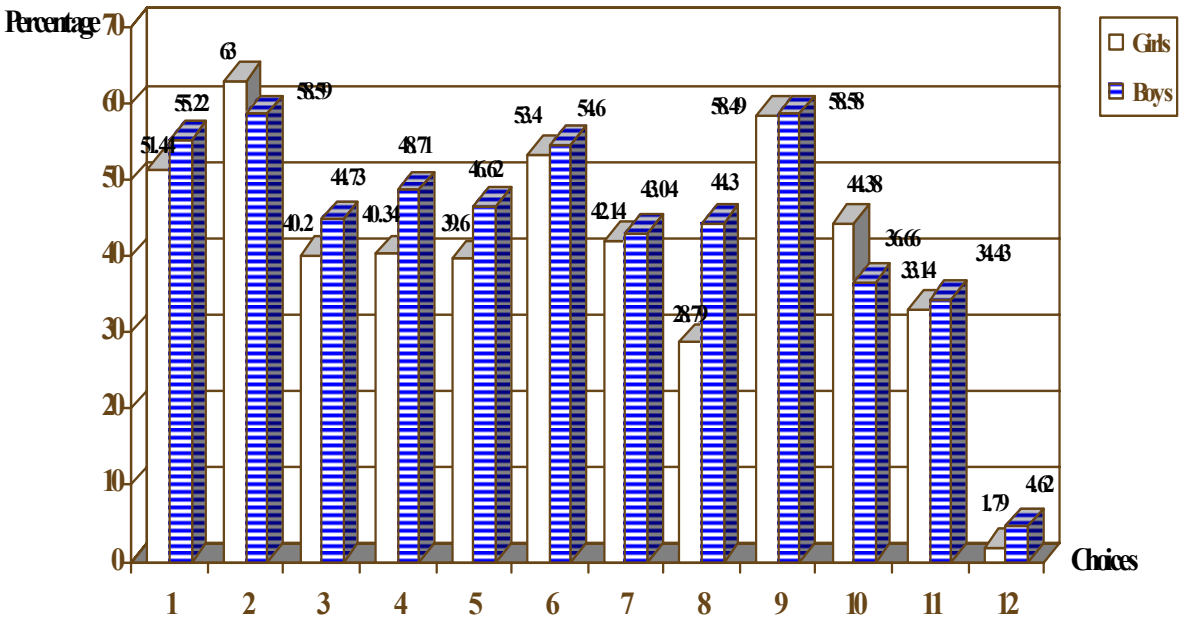
Most of the student teachers developed positive attitudes towards doing research during teaching practice. They have come to realize that doing classroom research can help them develop a wider perspective on how to view their classrooms. Moreover, they acknowledged that classroom research may help improve teachers' everyday practice.

Most importantly, student teachers in the project changed their attitudes towards learning from academic learning to life-long learning. Their paradigm on teaching and learning has been shifted into a new one which is seen a unification of their way of life and their learning. This also influences their educational values on their own contribution to society, the core values demanding for Thai society.

1.3.2 School students' recognition of learning experiences in the classrooms using Open-Approach method of teaching

In what follows, some of school students' learning experiences are illustrated according to items in the survey of about 1200 secondary school students in the above-mentioned 7 schools who experienced the open-approach method of teaching.

Figure 1 showing the responses to the item "Give the reasons why do you like doing activity in the classrooms?" (Select from the given choices)

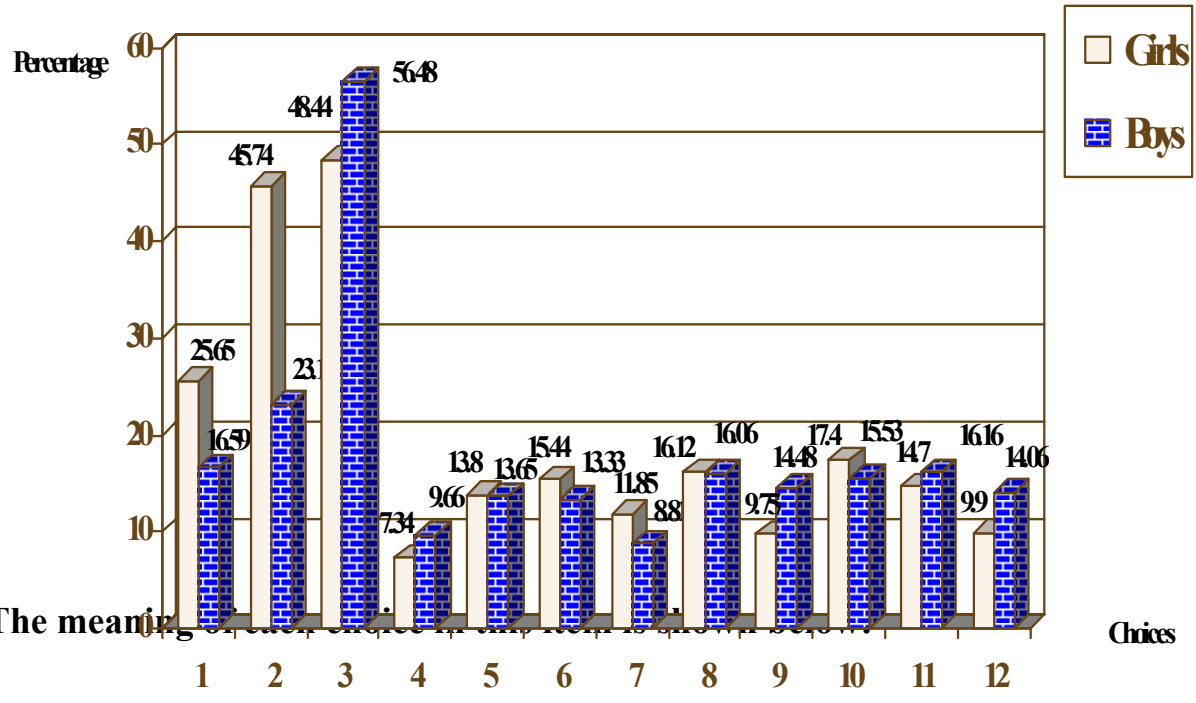


The meaning of each choice in this item is shown below:

1: More Active 2: More Thinking 3: More Playing 4: Use Art knowledge

5: Good atmosphere, friendship 6: Do something originally 7: Feel like independent time 8: Feel like be more valuable 9: Do real practice with given materials 10: Summarize some ideas by themselves (or by own group) 11: When think out, feel like "genius" 12: Feel not be boring.

Figure 2 showing the responses to the item "Give the reasons why you do not like doing activity in the classrooms?" (Select from the given choices)

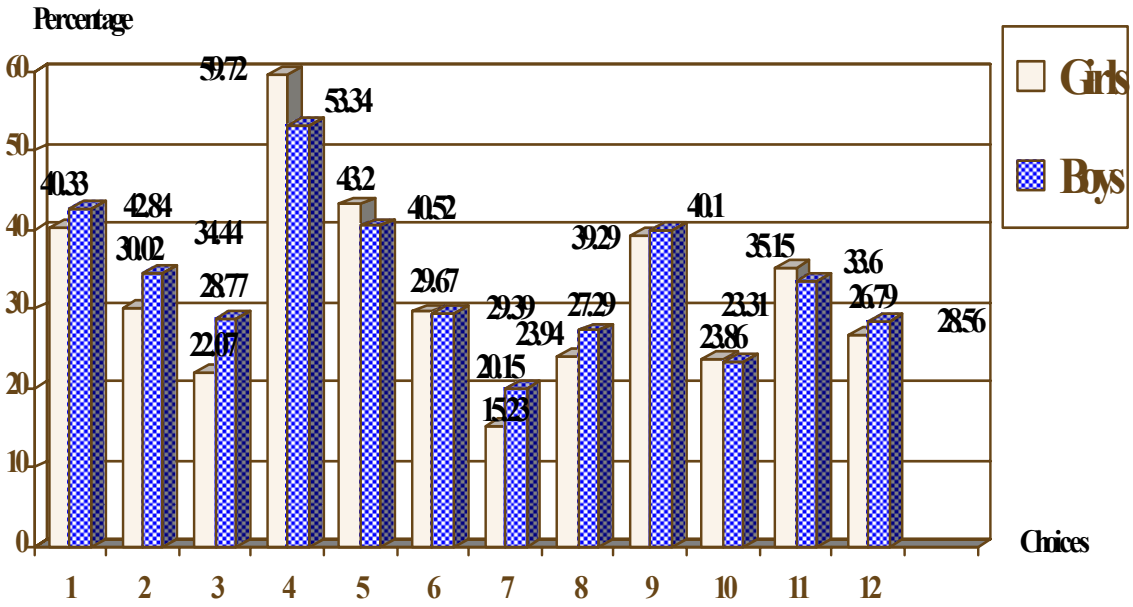


The meaning of each choice in this item is shown below:

1: Boring 2: Quite not understand questions or direction clearly 3: Feel that loudly Classrooms 4: Do not like working in group 5: Do not like someone in own group 6: Quite difficult activity 7: Do not know "to do for what" 8: Cannot conclude or connect ideas in activities 9: Feel that do not learn the same things as friends do in other classes 10: Do not know what to do to answer "the why how questions" 11: Teachers cannot observe all students 12: Time restricted.

Figure 1 and 2 shows a very high consistency that most of the school students like doing activity in the classrooms using Open-Approach method of teaching. The percentage of 2nd choice in figure 1 proved evidence that this kind of classroom activity enhancing the students to think than they used to be.

Figure 3 showing the responses to the item "Explain in what issues you change in positive way?" (Select from the given choices)

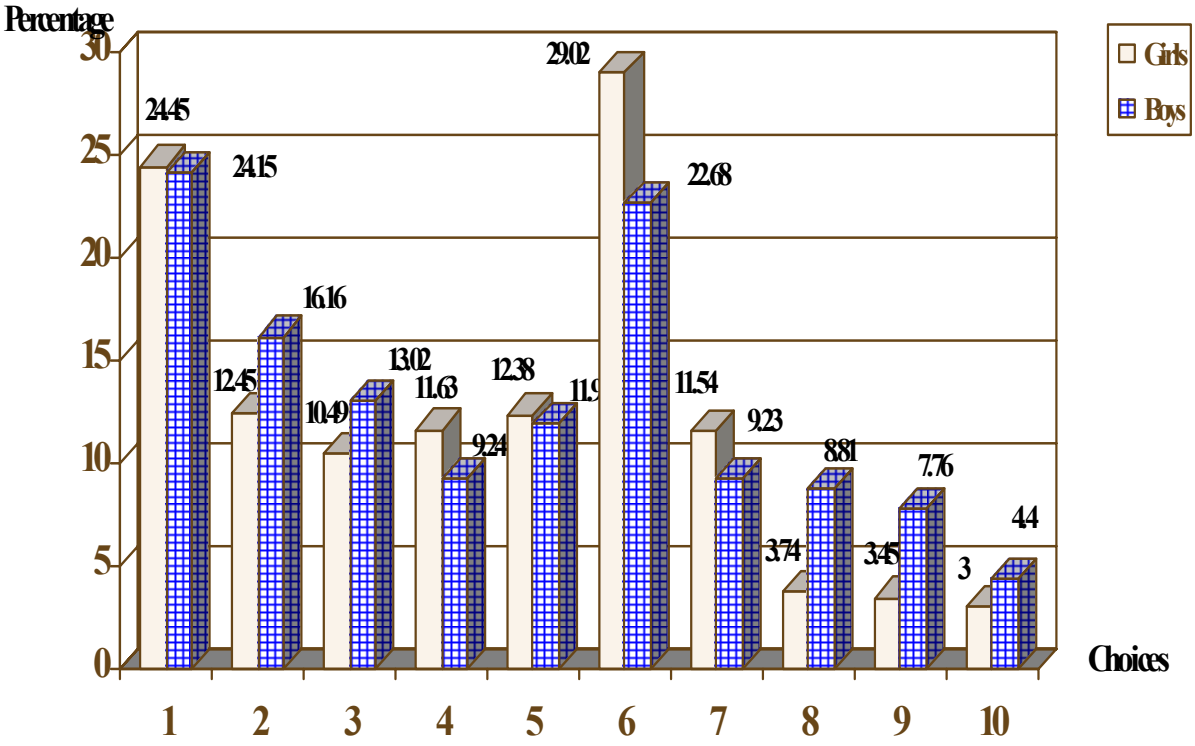


The meaning of each choice in this item is shown below:

1: More reasonable 2: More skillful in observation 3: More cooled-heart

4: Know how to work cooperatively 5: Dare to ask question 6: Dare to argue according to their own thinking 7: Dare to reject what they do not accept 8: Better communication with friends 9: Know how to solve problems in a variety of ways 10: More connected knowledge networking 11: More enthusiasm 12: Better achievement.

Figure 4 showing the responses to the item "Explain in what issues you change in negative way?" (Select from the given choices)



The meaning of each item is shown below:

1: Do not use the fullest ability or capacity 2: Lose confidence because of rejection of group 3: Friends or teacher dominate ideas 4: Inert 5: Being bored with maths than before 6: Tension and anxiety 7: Worse achievement 8: Quite show off 9: Feel not belong to group 10: Others (not friendship)

The responses to the items 3-4 mentioned above shows in what way the school students recognize their learning experiences. Choice 4 in figure 3 is the most interesting one. Nearly 60 percents of the school students learned how to work cooperatively. This situation sharply contrasts with the traditional classrooms in Thailand which mainly focusing on individual seat working.

From choice 9 in figure 4, since there is nearly 30 percents of the students who feel anxious, it is worthwhile to be concerned with this issue if we plan to expand the implementation of open-approach and lesson study in the future.

CONCLUSION

The project provides many ideas on professional development. The line between programs for student teachers and in-service teachers is blurred. It is worthwhile to conceive that programs for professional development should start in earlier years of teacher education programs. So far, lesson study approach started to have a great

influence on the reform of program for professional development in Thailand. The National Commission on Science and Mathematics Education incorporates the concept of lesson study into the framework on the development of science and mathematics education. The Faculty of Education at Khon Kaen University started implementing lesson study approach into a new 5-year program in the 2004 academic year. In the proposal submitted to the ministry of education, Thailand to establish Center of Excellence in Mathematics the concept of integrating open-ended approach and lesson study approach is put into the framework of professional development.

Khon Kaen University has also just started a training program for mathematics and science teachers from Lao PDR since 2002. This training program also implements the integrated open-approach teaching method and lesson study approach. This kind of professional development may create teacher networking among countries in the Great Mekhong Sub-region in the near future.

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APEC Conference

Specialist Session

Innovative teaching mathematics through Lesson Study

16 - 20 January 2006

APEC – Tsukuba International Conference
Specialist session (16 – 20 January 2006)

PROGRAM

16th January

17:00-17:50 Opening Ceremony
About the Goal of Specialist Meeting
Greetings, Explanation of Program and reconsidering

17th January

08:00 Rendezvous: in front of the IFIC.
Move to Attached Elementary School by walk and train.

09:00 Study Lessons in Attached Elementary School of the University of Tsukuba
Chair & Translation : Akihiko Takahashi, Makoto Yoshida, & Tad Watanabe
Presented by
Mr. Kozo Tsubota & Mr. Yasuhiro Hosomizu

Move to the Tokyo Campus of the University of Tsukuba by walk

13:30 Opening Discussion for Sharing Goals

14:00 Session 1: How can we discuss good practice? Chair : Marsigit

Developing good mathematics teaching practice through lesson study: a U.S. perspective
Akihiko Takahashi, Tad Watanabe & Makoto Yoshida (USA)

In search of good practice and innovation in mathematics teaching and learning: a Malaysian perspective
Lim Chap Sam (Malaysia)

Good practice of mathematics lessons in Korean
Park Kyungmee (Korea)

17:30 Closing of the opening session.

18th January

08:00 Rendezvous: in front of the IFIC.
Move to Attached Junior High School by walk and train.

09:00 Study Lessons in Attached Junior High Secondary School of the University of Tsukuba
Chair & Translation : Akihiko Takahashi, Makoto Yoshida, & Tad Watanabe
Presented by
Mr. Yutaka Ohneda & Mr. Akihiko Suzuki

Move to the Tokyo Campus of the University of Tsukuba by walk

13:30 Session 2: Professional development & project through Lesson Study

Chair : Tran Vui

Good teaching in Hong Kong
Frederick Leung (Hong Kong)

Pursuing good practice of secondary mathematics education through lesson studies in Indonesia
Marsigit (Indonesia)

Good practice in mathematics teaching and teanching development
Yeap Ban Har (Singapore)

Development of effective lesson plan through lesson study approach: a Thai experience
Maitree Inprasitha, Suladda Loipha, Ladda Silanoi (Thailand)

17:30

19th January

8:30 Session 3. Lesson Study for Teaching Contents Chair : Le Hai Khoi

Strategies for addition and subtraction of whole numbers extended to number to number sentences involving fractions and decimals
Max Stephens (Australia)

Beginning the study of the additive field
Grecia Ga'lvez (Chile)

On the enhancement of creative & independent awareness of primary school pupils
Le Hai Khoi (Viet Nam)

Helping students develop and extend their capacity to do purposeful and worthwhile mathematical works
Tran Vui (Viet Nam)

Session 4. Lesson Study for mathematical activity and learning how to learn

Chair : Okubo, Kazuyoshi

Good mathematics teaching practices – in the making: a Philippine experience
Soledad A. Ulep (Philippines)

Session 5. Developing and elaborate the document as for resume and recommendation.

Chair : Maitree Inprasitha & Isoda, Masami

20th January

8:30 Session 6. For next Thailand Conference Chair : Suladda Loipha

11:30 Closing

Reflecting on Good Practices via VTR

Based on a VTR of Mr. Tanaka's lesson 'How many blocks?

Masami ISODA, University of Tsukuba

The focus of the Tokyo and Thailand meetings are to share ideas on good practices from participants and to structure, develop and review the product using VTR for teacher education and reform movement in Mathematics Education. The Tokyo meeting focuses on developing a format for sharing good practices. This paper proposes the following special format: a) Short summary of the lesson with emphasis on major problems in the lesson, b) Components of the lesson and main events in the class, and, c) Possible issues for discussion and reflection with teachers observing the lesson. Based on this format, good practices may be identified and explained. This procedure strengthens the proposal that the function of the VTR is more than merely the re-production of good practices.

Key ideas for using VTR in relation to the Lesson Study

Japanese Lesson Study has been developed from 1873 (See Appendix A). There are various kinds of usage but, currently, in narrow meaning, a Lesson Study is divided into three parts: a) planning the lesson, b) the observation part, and, c) the discussion and reflection part. These parts may be summarized by the words: *plan, do* and *see*. The lesson study is usually done through the collaboration of teachers. *Plan, do* and *see* can be done by a teacher but as we know, it is not necessary to begin from *plan* in daily practice. Indeed, many teachers do not write their lesson plan everyday. A teacher's knowledge is usually developed through the order of *do, see, plan*. In any case, the part of reflection or *see* is a key component of the lesson study. In the process, teachers participate in the study lesson and reflection, and learn ideas for their next lesson.

When we use the VTR, we also begin from the lesson observation but the VTR itself already loses many dimensions, parameters and context because the program is prepared (recorded) from the perspective of the recorder's and VTR editor's eyes only. Through the observation of the VTR, we learn things and apply these in the next activity. It is also the process of *do, see* and *plan*, or observation, reflection and planning. From the phrase 'Reflecting on the Good Practice (RGP) via VTR', I would like to propose these processes (*do, see* and *plan*) in teacher education, reform movement in mathematics education and mathematics education research.

The phrase or idea of RGP, rather, focuses on a methodology for innovation in mathematics education. The TIMSS Video Tape Study illustrated this very well. Even if

people do not know the lesson in the US, Germany and Japan, they absorbed some messages from the short VTRs excerpted from the lessons and later discussed. What the mathematics educator does is develop the lesson in the form of a research.

An example of good lesson and a way of description

The VTR used is in the following website:
http://e-archives.criced.tsukuba.ac.jp/en/result_data.php?idx_key=1034
http://www.criced.tsukuba.ac.jp/math/teaching-material.html

Exploring Japanese Mathematics Lesson	.wmv
-For sharing key ideas-	(short ver.)(53.3MB)
	.wmv
	(long ver.)(28.1MB)

The lesson in the VTR is titled ‘How many blocks?’ The longer version contains an 8 minute edited lesson and almost 5 minutes of comments by Max Stephens.
In the context of developing usages of the VTR, Abraham Arcavi (to appear) developed a formatted description as follows:

a) Short summary of the lesson showing the major problem areas of the lesson.
b) Components of the lesson and main events in the class.
c) Possible issues for discussion and reflection with teachers observing the lesson.

The structure of this format is a good way of describing the use of the VTR in the context of teacher education for following reasons:
Viewing lessons via the VTR takes time. A short summary is necessary to grasp the contents quickly. Components of the lesson and events are useful in order to understand the contents quickly. If we do not have this background, we have to observe the VTR again and again to understand its contents clearly. Issues for discussion and reflection may be resorted to when using the VTR for teacher education.
The description of the VTR in this format is the format of RGP. The description does not need to be done by a VTR editor. On the contrary, it may be better that it is done by others because in the case of an editor, the description may not be the issue for discussion and reflection by others but may be fixed on his issues. There is actually a diversity of possible issues depending on the users’ perspective, ideas and context.
The Appendix B is an example by Aida Yap (to appear). Please refer to the VTR and read the document she prepared. She did not have a chance to look at the original lesson and edited original VTR but she described very interesting issues clearly. I compared her description with my ideas after I observed the original lesson and edited the VTR.

Context, De-contextualization and Re-Contextualization

RGP focused on good practices, good lessons or innovative lessons for the reform of mathematics education. It is accepted as a model for innovation of mathematics education by the person who selected it. The word 'reform' is usually defined by the problem and the aim of mathematics education based on personal recognition or by reference to same policy documents. The research based on evidences for making hidden valuables clearer is important. On the other hand, some things are not easy to reform even if we find them. For example, we may find that a teacher's belief is an obstacle to change the lesson but teacher's beliefs are not easy to change. Even if we try to be constructive, there are a number of constructive approaches depending on the number of teachers. On the other hand, RGP does not fix the word or value from the beginning. It tries to share the example as a model of good practice or lesson and we can use it what is good, how it is good, for doing it what is necessary conditions and how it was developed..

In any case, the edited VTR is not the same as the original lesson. It is de-contextualized through the process of recording and re-contextualized through editing. The user and observer of the VTR is not the editor. Through questioning by the users or observer of the VTR for reflection, it is re-contextualized.

Indeed, the interesting questioning by Aida Yap in the Appendix B is not the same as the original context and editor's context. She discovered aims of the lesson but she did not describe some special meanings of the original lesson study done by Hiroshi Tanaka. By reforming the Japanese standard, we lost the context of the study on solids at grade 1 in elementary level. Thus, she introduced solids in exploring the context of counting. When Japanese observed the study lesson at the recorded year, we observed it on the context of curriculum development such as how pupils can explore the solid on the context of counting and calculating problem setting on the curriculum standards. In the original context, this is a most important reason for explaining why this lesson study is a good practice.

Aida Yap discovered most of editors' perspectives but could not question some parts. Editors cut so many interesting parts for focusing on translation among representations of solids, pictures and expressions. She discovered them but she did not ask what part the editor found the most interesting in the lesson.

As the editor of VTR, I would suggest adding the question "Why did a pupil ask 'Did we study mathematics today?' at the end of the lesson?" I would do so because, as explained, the students did not have experience in exploring solids and that they

believed studying a new calculation is the lesson of mathematics. At the same time, it implies that it was an enjoyable activity for them (the students) which they associated as being the same as playing blocks at the beginning of the lesson. The limitation of the lesson, the inability to introduce new contents on solids, will be taught in the upper grade. In Japan, many teachers have a strong belief that we could not teach beyond the standards even if it is permitted to do so.

De-contextualization in the process of the description of the VTR depending on the format is unavoidable. And in the context from other economies, some issues are not significant. From the different ways of questioning we can know the different perspectives in mathematics education as well as commonality of perspectives. Ms. Yap's ways of questioning well imply that the curriculum differences are some issues for RGP via VTR.

How can we develop good teacher's perspective on teachers through the VTR?

Questioning helps us re-contextualize the VTR. In the process of pre-service teacher education, it is important to develop teacher's perspectives. Learning to listen is a key word for this approach. In the case of Japan, lesson study usually begins by developing a lesson plan. At this stage, teachers solve and pose problems from students' perspectives. By analyzing problems, teachers develop good ways of questioning. For writing the description of the VTR, it is very important to ask why? Why did students say this? Behind their words, there must be so many kinds of ideas. Why did the teacher say that? Through these questions, we can better know and understand the hidden features of the lessons being observed through VTR. Then, it is very important to add the format such kinds of descriptions from the view points of original lessons but even if we add descriptions we do not needs to follow because re-contextualization is done by VTR users.

Acknowledgement

In Appendix, I used the paper by Aida Yap (to appear) in the book edited by Masami Isoda and Max Stephens. I would like to express my special thanks for her contribution.

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A Brief History of Mathematics Lesson Study in Japan: "Where did Lesson Study begin, and how far has it come?"

Lesson Study began in the late 19th century with class visits designed to allow the study of group instruction.

1. From Individualized Instruction to Group Instruction: Studying Teaching Methods

Under the seclusion policies and class system that characterized the Edo period for about 260 years prior to the installation of the new Meiji government in 1868, literacy (and numeracy) education was available to commoners through *terakoya*, or temple schools, that had opened up autonomously around the country. Commerce thrived and the class system gradually collapsed during this period of seclusion, and by the late Edo period, individual knowledge and skills were highly regarded in the recruitment of workers. Due to the widespread emergence of temple schools, to which parents could voluntarily send their children, the literacy rate at the end of the Edo period was 43% among males and 10% among females, even then making Japan one of the most educated countries in the world. Individualized instruction was the common teaching method employed.

In 1872, the Meiji government issued the Education Code and at the same time established a teachers' school (normal school) in Tokyo (forebear to University of Tsukuba). With the goal of disseminating Western scholarship, the government invited foreign teachers to teach Western subjects. The foreign teachers introduced

the concept of group teaching, a style then still rare even in the West, into the teachers' school (Fig. 1). The Japanese teachers and students, who were familiar only with the individualized instruction model in which subjects were taught individually based on the academic abilities of the student, learned not only the contents of the subject, but also methods of teaching by observing their teachers' behavior.

Textbooks created by foreign teachers at the teachers' school contained drawings of students raising their hands to answer questions posed by the teacher, as shown in Fig. 2. It contained the question "How many students are raising their hands?" This foreign teacher wrote a textbook that teaches instruction methods as well as mathematics at the same time. The group instruction model implemented at the teachers' school in Tokyo spread to other teachers' schools around the country. Due to financial difficulties, the new government closed down all the teachers' schools except the ones in Tokyo around 1880.

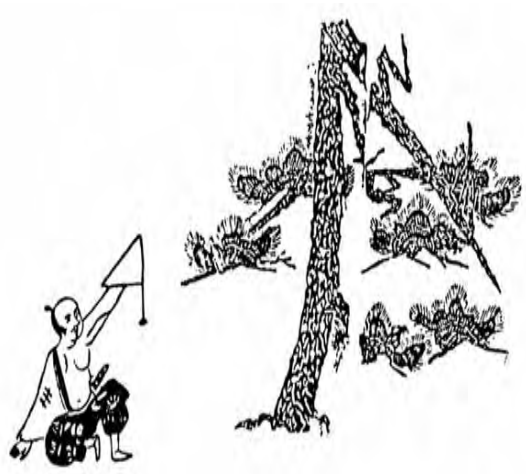
Nonetheless, in the decade while the schools were open, the practice of group instruction was disseminated around the country by graduates of the teachers' schools and by scroll pictures (Fig. 1) and textbooks (Fig. 2).

1868 New Meiji Government
Imperial government, country opening,
Westernization policies

1872 Education Code issued, teachers' schools established
How is teaching done in a classroom?
How does group instruction work?

Pestalozzi: Experience-oriented teaching methods

Fig. 1 Shift from the curriculum and teaching methods of the *terakoya* (temple schools) to those of new types of schools.



"How tall is the tree? "

Illustration from *Jinkoki*, a mathematics textbook from the Edo period.

"How many people are raising their hands?"

Illustration from an elementary mathematics textbook in 1873.

Fig. 2 From textbooks that allowed students to study numeracy at their own discretion, depending on their needs, to textbooks designed to allow students to simultaneously study teaching methods.

2. Dissemination of the Lesson Study Practices through the Elementary School Attached to the Tokyo Teachers' School.

In the 1880s, study on group instruction and its dissemination reached new heights as overseas study missions began returning to Japan. Mission delegates, who had been teachers at the teachers' school before their departures, were invited to become teachers at the elementary school attached to the teachers' school after their return, and a book on the Pestalozzi's teaching method was published. Even back then, this book contained comments on teaching materials, as well as instructions for conducting class observation and holding critique sessions. At the instruction of the Ministry of Education, these teaching methods were implemented throughout Japan as a model. Open classes were held to encourage the proposal of new teaching methods and teaching curricula, producing the first interactive Lesson Study study groups initiated by the government.

Fig. 3 shows one of the national teachers' training conferences, which have been held since the Meiji period.

3. Development and Dissemination of Teaching Methods Learned through Lesson Study

As the country grew wealthier, it became possible for anyone to attend and graduate from elementary school. In the 1920s, new teaching methods based on the educational philosophy of scholars like John Dewey launched an era in which non-government-attached-school teachers began proposing their own teaching methods. At this time, a new teaching method was proposed for enhancing peer learning (see Fig. 4). It allowed students to come up with their own study questions,

discuss with one another whose question they wanted to research, and then go about researching the selected question. This set the stage for the emergence of teaching methods that focus on problem-solving, which today are globally recognized as models of constructivist teaching. Teachers' unions were launched after World War II, and Lesson Study by involved teachers led to heated debates. These classes also came to be used for launching futile ideological opposition. Teaching methods focused on problem-solving, which recognized the limitations of what already known and tried to produce new knowledge, were able to achieve success in spite of having to overcome the conflicts and other challenges. This was possible because visiting teachers were exposed to classes conducted for observation, and were impressed by seeing the students learning by themselves through problem solving.

Now, problem-solving approach is well known as a major way of teaching mathematics in Japan.

A class with 100 observing teachers.

A group of 1,200 teachers observe a class and class review session on an auditorium stage.

Fig. 3 National Training Conference for Teachers at the Elementary School Attached to the University of Tsukuba, held since the Meiji period.

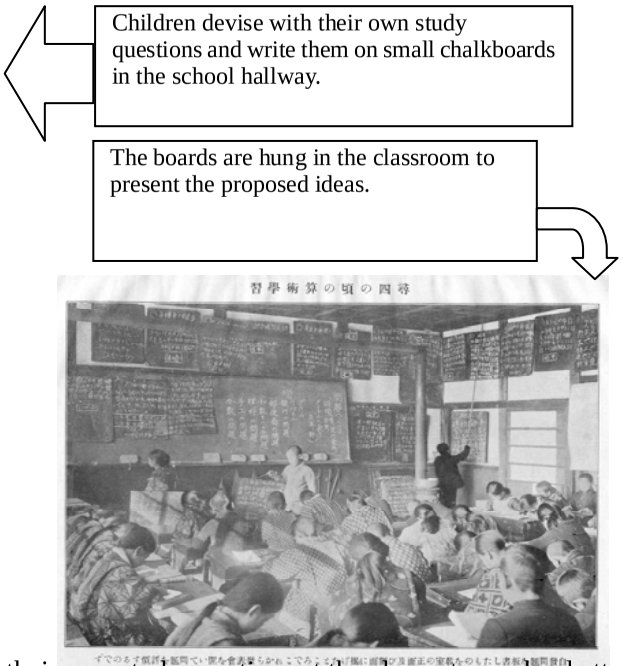


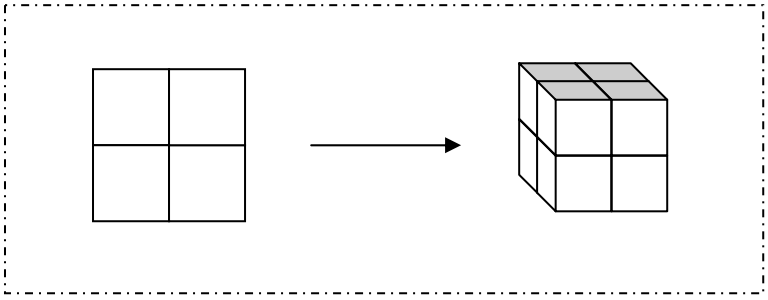
Fig.5 Study is conducted on how to teach students to develop their own study questions at the elementary school attached to Nara Women's higher normal school around 1920.

Appendix B

"How many blocks?"
A first grade mathematics lesson

Aid Yap, University of Philippines

The topic of this first grade lesson is on determining the number of blocks in the pile wherein some of the blocks are hidden from one's view. The main objective of this lesson is to engage students in visualizing the number of blocks in the pile and explaining how they got their answers. In order to determine the number of blocks in the pile, the students have to rely on their visualization skill.



Components of the lesson and main events in the class:

- A pile of blocks and a camera are hidden from the students. The front view of a pile of blocks is shown on the television screen. The teacher then asked the students to determine the number of blocks in the pile. This part of the lesson encourages the students to guess because showing the front view of the pile of blocks is quite deceiving. Most of the students answered 4 blocks, which was not surprising at all.

- The teacher afterwards positioned the camera at a different angle so that another view of the pile of blocks is shown on the television screen. As before, the teacher asks the students to determine the number of blocks they think there are in the pile.

- A drawing of what was shown on the television screen was posted on the board. The teacher distributed a worksheet to each student. The worksheet contains the same drawing as the one posted on the board. The students were then asked to

write their formulation and answer in the worksheet.

- Students came up with different mathematical formulations such as 4 + 4, 3 + 2 + 3, 1 + 3 + 4, 4 + 3 + 1, and 2 + 2 + 2 + 2. Some students were asked to explain their work in front of the class.

- Towards the end of the lesson, the teacher brought out 8 big blocks and arranged them in a pile similar to what was shown in the drawing. The students were asked to come closer to the front so that they could clearly see the pile of blocks. The teacher repeated the explanation of some students using these blocks.

Possible issues for discussion and reflection with teachers observing this lesson:

What may be the goals of this lesson?

By showing the front view of the pile of blocks and writing on the board the question that students need to answer (How many blocks are there in the pile?) the teacher sets the goal of the activity. The teacher is not actually interested on whether the students come up with the correct number of blocks in the pile but rather on the students' way of thinking in getting the number of blocks in the pile.

How can we characterize the mathematics of this lesson?

Visualization skill is a very important skill that any student must possess. Thus, giving problems that help develop the visualization skill of the students is really important even at this very early stage in elementary mathematics. Encouraging students to explain or defend his/her answer is really more important than the answer itself. In this way, the teacher would be able to discover student's mathematical thinking and possible misconceptions that the student may have. Corrections on the erroneous ways of thinking of the student may then be made accordingly.

How does the teacher view his students?

The teacher is always challenging the students all the time to imagine the number of cubes in the pile. Never did the teacher say that the answer given by the student is correct or not. It is evident that the teacher is not after the answers given to him by the students but rather on the thinking or reasoning behind each answer. The teacher feels confident that even at this early age students would be able to show evidence of their visualization skill.

What are the characteristics of the classroom management of this teacher?

The teacher made use of a combination of strategies to get students attention all the time. He writes, explains, poses problems/questions, and process students' answers. The use of television to enhance his instruction was really a good idea to challenge the students to think. During the lesson proper, the teacher showed expertise in handling the discussion. After a student presented his/her work, the teacher always followed-up student's explanation.

It is very evident that the teacher was able to capture students' attention through the activities he presented. Students really enjoyed the hands-on and minds-on activities given to them by the teacher. There was never a lull period during the discussion.

Is there more mathematics stake in this problem of which the teacher should be aware of?

The teacher obviously attained his intended goal for this lesson. It would be interesting to find out the reasoning behind the other mathematical formulations made by the students. It is farfetched to expect these first graders to come up with mathematical formulations involving multiplication.

What may be the learning outcomes and the follow-up for such lesson?

From the video, it is clear that the students were able to come up with different ways of counting the number of blocks in the pile. In all these mathematical formulations, the visualization skill of the students is being challenged. It would be very worthy of note to find out if students can deal with counting the number of blocks in the pile containing more than 10 blocks or when there are more blocks hidden from the students view.

DEVELOPING GOOD MATHEMATICS TEACHING PRACTICE THROUGH LESSON STUDY: A U. S. PERSPECTIVE

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Although there is no consensus on what constitutes good mathematics teaching practice in the United States, a recent document published by the National Research Council (NRC) offers a vision that might be acceptable to the various stake holders. The NRC document considers teaching as an "interaction among teachers and students around content." Lesson study may play a significant role in developing and spreading good mathematics teaching practices that are in alignment with the vision presented in the NRC document. In this paper (and accompanying video of a lesson), we will discuss some specific features of good teaching practices and how lesson study may contribute to the development of such practices. We will conclude the paper with a brief discussion of future research that may be fruitful.

INTRODUCTION

As the participants of the APEC conference may be aware, recently there has been a lively debate about mathematics education in the United States. This debate, often called the "Math Wars," has largely focused around the new mathematics curricula developed to implement the *Standards* (National Council of Teachers of Mathematics [NCTM], 1989, 2000). These curricula de-emphasize teacher-telling as the primary mode of instruction. Rather, they organize their units around student investigations and discussions, to help students develop conceptual understandings and their own procedures, often very different from conventional procedures. Furthermore, these curricula emphasize the integration of mathematics, both within and beyond the field of mathematics. Thus teaching practices necessary to successfully implement these curricula are significantly different from that of direct instruction. Critics, such as the group called Mathematically Correct, argue that such an approach disadvantages significant segments of the student population. Thus, it should be clear that there is no consensus in the United States on what constitutes good mathematics teaching practice. However, a recent publication, *Adding It Up* (National Research Council, 2001), seems to offer a possible vision of good practice that may be agreeable to all sides of the

current debate for it is based on the work of the Mathematics Learning Study Committee consisting of people from different viewpoints.

GOOD PRACTICES

One cannot discuss good or effective instructional practices without considering the goals of instruction. In *Adding It Up*, the authors present the notion of mathematical proficiency consisting of the following five interwoven strands:

- *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *Procedural fluency* – skill in carrying out procedures flexibly, accurately, and appropriately
- *Strategic competence* – ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

(NRC, 2001, p. 116)

Thus, good teaching practices should promote the development of these strands of mathematical proficiency. Moreover, since these strands are "interwoven and interdependent" (p. 116), good teaching practices cannot focus on just one or two of these strands.

The document considers teaching as "interactions among teachers and students around content" (NRC, 2001, p. 313). In particular, they adapted the teaching triangle model developed by David Cohen & Deborah Ball shown in Figure 1.

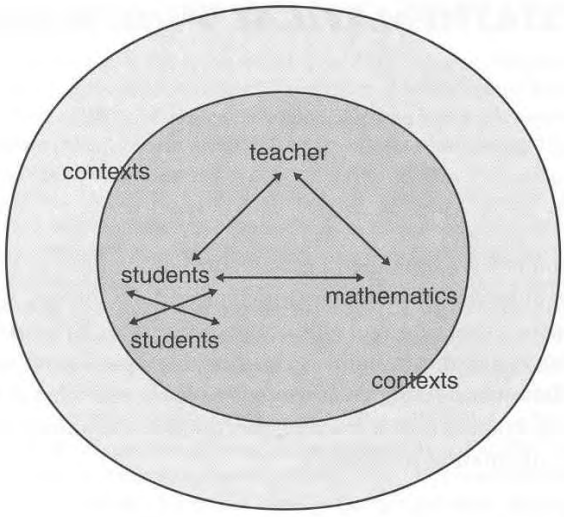

Figure 1: This model shows that mathematics teaching as the interaction among teachers, students, and mathematics, all taking place in contexts (NRC, 2001, p. 314).

Therefore, good mathematics teaching practices must go beyond simply what teachers do as they teach mathematics lessons. In particular, the document suggests,

> High-quality instruction, in whatever form it comes, focuses on important mathematical content, represented and developed with integrity. It takes sensitive account of students' current knowledge and ways of thinking as well as ways in which those develop. Such instruction is effective with a range of students and over time develops the knowledge, skills, abilities, and inclinations that we term mathematical proficiency (p. 315).

In the following section, we will illustrate some features of good mathematics teaching practices from a Grade 6 lesson taught during a lesson study open house.

AN ILLUSTRATION: RESEARCH LESSON ON AREA OF TRIANGLES

The following example is from one of the research lessons developed by a lesson study group in the U.S.. Unlike typical lesson planning, a group of teachers spent a couple of months to plan the lesson, studying the specific mathematics topic intensively and investigating the available resources, including a translated Japanese textbook. One member of the group, Mr. Jackson, was selected to teach the lesson. In order to accommodate a large number of observers, the lesson was conducted in a gymnasium.

The teacher opens the lesson by having selected students read what they wrote about what they learned in the previous day's lesson. This interaction clearly reminds the students some of the important ideas they learned in the previous day's lesson. However, in addition, this interaction illustrates how Mr. Jackson is considering his students' current thinking in setting up the main task for the lesson. Prior to the lesson, he selected which students to call on and in what order. By sequencing the students' comments (and their work from previous day's lesson), Mr. Jackson is able to provide a cohesive summary of the important mathematical ideas from the previous lesson, instead of a collection of haphazard recollections by randomly selected students, possibly leaving out some important ideas.

Another important point to notice in this opening segment is that, by integrating the students' own ideas in the lesson, Mr. Jackson communicates to his students that, in this classroom, their own ideas and methods are valued. Such an expectation has been identified as an important feature of a classroom that is set up as a community of learners (Hiebert et al., 1997).

This emphasis on students' own ideas and methods are again stressed in the next segment of the lesson, where Mr. Jackson poses the main task for the lesson – finding the area of a triangle by changing its shape. However, the first thing Mr. Jackson asks his class to do is to write down their ideas on how they might approach this task. By posing this question, Mr. Jackson communicates to his students that what is valued in this lesson is not just the answer, i.e., the area of the particular triangle, but also various ways students can determine the area. Furthermore, by having students share some of their ideas, Mr. Jackson provides an opportunity for those students who may be unsure about the task to consider ideas that they may pursue.

As the students begin their investigation, they were provided with many copies of the triangle on papers of different colours. Students are expected to cut and paste the triangles to illustrate their method clearly. The decision to use this particular set of materials in the lesson was not made lightly. During the post-lesson discussion, Mr. Jackson stated that he and his colleagues have explored a variety of materials, including the use of geoboards. However, the planning team felt the actual experience of cutting and re-arranging the given figure would provide an important foundation for the students to make sense of the area formula, which was the eventual goal of the unit. This type of careful consideration of instructional materials in light of the students' current understanding and the mathematical goals is another indication of effective mathematics teaching practices.

As the students engage in their investigations, they are free to choose whatever method that makes sense to them. As they experiment with their ideas, sometimes they attempt methods that are not productive. However, they are given the opportunity to determine whether or not their ideas are correct based on logical necessity, not because their teacher says so. Granting students such autonomy is another feature of a classroom as a community of learners (Hiebert et al., 1997).

In the next segment of the lesson, Mr. Jackson has his students share their ideas. As he did at the beginning of the lesson, Mr. Jackson carefully sequences students' ideas. By selecting and sequencing students' ideas intentionally. Mr. Jackson attempts to match students' diverse thinking processes with the development of a particular mathematical idea, illustrating interaction between the teacher, the students, and mathematics.

Furthermore, student work is clearly displayed on the blackboard, both their work with paper arrangements and mathematical expressions. In many so-called reform mathematics lessons in the United States, teachers often ask students to share their solutions publicly. However, too often, the sharing of students' solutions becomes the end of the lesson. That was not the case in Mr. Jackson's lesson. Perhaps the most important segment of the lesson is the next phase of the lesson where the teacher poses some questions to further analyse the ideas and methods shared by the students. In this particular lesson, Mr. Jackson asks students to sort the variety of methods into two types – those which transformed the given triangle to another shape without changing the area and those which created another shape by doubling the area of the given triangle. In the lessons to follow, the class will formally derive the area formula for triangles, but the experience gained in this lesson is an important foundation in understanding why the formula includes the division by 2.

Mr. Jackson ends the lesson by providing a summary of the important ideas discussed in the lesson. In his summary, he connects the day's lesson with the previous lesson by referring back to the work shared at the beginning of the lesson. This segment illustrates once again how the selection of the ideas to be shared at the beginning was intentional. Moreover, making this connection communicates to the students that mathematics learning should be based on what they have learned previously.

This particular lesson is by no means perfect. We do not claim that all students in the lesson learned everything discussed in the lesson completely. However, we offer this lesson as an illustration of good teaching practice that attempts to address the five strands of mathematical proficiency in an integrated manner. What is important to consider here, however, is that Mr. Jackson was not born a master teacher. He openly admits that his teaching was very different as recently as five years ago. He learned many of the features we discussed above through his participation in lesson study at his school. Furthermore, what Mr. Jackson learned through his participation in lesson study was not his alone. Other teachers who participated in lesson study also developed deeper understanding of good mathematics teaching practices. In the following section, we will discuss how lesson study may promote the improvement of mathematics teaching.

ROLE OF LESSON STUDY IN DEVELOPING GOOD TEACHING PRACTICE

As the participants of the APEC conference know, lesson study is the primary mode of professional development in Japan. Lesson study has played an important role in professional development in Japan since the beginning of the public education system in Japan more than a hundred years ago. One of the reasons for this popularity might be that lesson study provides Japanese teachers with opportunities to do the following: a) make sense of educational ideas within their practices; b) change their perspectives about teaching and learning; c) learn to see their practices from the child's perspective; and d) enjoy collaborative support among colleagues. For example, one Japanese teacher said:

> It is hard to incorporate new instructional ideas and materials in classrooms unless we see how they actually look. In lesson study, we see what goes on in the lesson more objectively, and that helps us understand the important ideas without being overly concerned about other issues in our own classrooms (Murata & Takahashi, 2002).

Why is lesson study so appealing to so many US researchers and educators? We think lesson study has certain characteristics that set it apart from the typical professional development program in the U.S., and that these unique characteristics are what makes lesson study so popular. These characteristics are described below.

First, Lesson Study provides teachers the opportunity to see teaching and learning in the classroom in a concrete form. This is due to the fact that lesson study guides teachers to focus their discussions on planning, implementation, observation, and reflection of classroom practices. By looking at actual practices in the classroom, teachers are able to develop a common understanding or image of what good teaching practice entails, which in turn helps students understand what they are learning.

Another unique characteristic of lesson study is that it keeps students at the heart of the professional development activity. Lesson study provides an opportunity for teachers to carefully examine the student learning and understanding process by observing and discussing actual classroom practices. Understanding student misunderstandings is

often examined in the process of observing and discussing the lesson. This also contributes to helping students construct their understanding. A third characteristic of lesson study is that it is teacher-led professional development. Through lesson study, teachers can be actively involved in the process of instructional change and curriculum development. Lynn Liptak, a retired principal at Paterson Public School No.2, Paterson, NJ, who has been implementing lesson study for over 4 years, contrasted lesson study with traditional professional development in the U.S., as summarized in Table 1.

As can be seen from Table 1, lesson study is teacher-led professional development where all the participants reciprocally learn from each others' experiences. In addition, the collaboration helps reduce isolation among teachers and helps to develop a common understanding of how to systematically and consistently improve instruction and learning by the school as a whole. Moreover, lesson study is a form of research that allows teachers to take a central role as investigators of their own classroom practices and become life-long autonomous thinkers and researchers of teaching and learning in the classroom.

Traditional	Lesson Study
Begins with answer	Begins with question
Driven by outside "experts"	Driven by participants
Communication flow:	Communication flow:
trainer → teacher	teacher ↔ teacher
Hierarchical relations between trainer & learners	Reciprocal relations among learners
Research informs practice	Practice is research

Table 1: Contrast between lesson study and traditional U.S. professional development (By Liptak, as included in Lewis, 2002, p. 12)

It is because of these features that lesson study has the potential to influence the quality of mathematics teaching practices in the United States, and elsewhere. Lesson study offers opportunities for participants to critically evaluate teaching practices. Such critical dialogues may take place during the planning period among the planning group members, or they may take place during the post-lesson discussion. In either case, these dialogues take place in the context of actual lessons, developed carefully and intentionally. Through such critical evaluations, lesson study provides a concrete image of effective instructional practice. Hiebert, Gallimore, and Stigler (2002) suggested that lesson study might be a potentially useful way of sharing good teaching practices. However, we suggest that lesson study is not only a useful tool to disseminate effective teaching practices but also a powerful mechanism to develop such practices.

As we noted at the beginning of this paper, the mathematics education community in the United States is in the midst of debate. Although people may disagree with each other, they are all concerned about students' mathematics learning. In order for us all to learn from these debates, we need to make sure that the debates are deeply rooted in the actual teaching of mathematics, and lesson study offers a systematic forum where such debates to can take place.

FUTURE RESEARCH

Just as mathematical proficiency involves interwoven and interconnected strands, good teaching practices, that is, teaching practices that promote mathematical proficiency, also involve interrelated components. *Adding It Up* lists the following five components for proficiency in the context of teaching:

- *Conceptual understanding* of the core knowledge required in the practice of teaching;
- *Fluency* in carrying out basic instructional routines;
- Strategic competence in planning effective instruction and solving problems that arise during instruction;
- Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them; and a
- *Productive disposition* toward mathematics, teaching, learning, and the improvement of practice (NRC, 2001, p. 380).

Although lesson study seems to offer a promising pathway to an improvement of mathematics teaching practice, there are yet many questions that need to be addressed through further research. In particular, if we are to accept the notion of mathematics teaching *proficiency*, we must investigate how teachers develop such proficiency. One important question that needs to be addressed is the relationship between teacher knowledge and teacher practice. What knowledge do teachers draw on to teach mathematics more proficiently? How do they develop such knowledge? Other questions will have to address the effectiveness of educational policies in promoting proficient teaching practices. What support must school systems provide to practicing teachers so that they can continue to develop their proficiency? What are the appropriate responsibilities of teacher education institutions in preparing the beginning teachers?

As we engage in this research in the future, we should also keep in mind the value of cross-system research. Although teaching occurs in contexts, and our contexts vary significantly, our future research can nevertheless inform each other. One recommendation for improvement of mathematics teaching practices offered in *Adding It Up* states that professional meetings should be used for "more serious and substantive professional development" (p. 430). Likewise, when mathematics education researchers throughout the world come together, we should use those

occasions for sharing and planning further collaborative efforts to improve mathematics teaching practices everywhere.

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In Search Of Good Practice And Innovation In Mathematics Teaching And Learning: A Malaysian perspective

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Introduction

This paper begins with a description of good practice of mathematics teaching as perceived by the Malaysian perspective. The sources of discussion include those from the school curriculum, local related research studies and the practicing mathematics teachers. The second part of this paper will focus on a small research project that promotes practices of good mathematics teaching through the Lesson Study model. Issues and challenges that we faced in introducing Lesson Study as an innovative model of teacher professional development will be discussed. A video clip of one of the lesson planned and taught during one of the Lesson Study cycle will also be displayed for further discussion.

What is a description of good practice from the Malaysian lenses?

There is yet to have consensus of a single definition of "good practice". Mohd Majid Konting (1997) proposed that information about good practice may come from various sources: theory, research, curriculum planners as well as expert teachers. Professor Nerida Ellerton (2003) in her summary of best practices and innovations in the teaching and learning of mathematics among the 6 represented APEC economies (namely Australia, Japan, Korea, Malaysia, Singapore and United States) listed four guiding principles for defining best practices:

a) Best practice is not the same the world over
b) Best practice needs to be developed in the school level
c) Models of best practice need to be shared
d) Best practice needs to be valued at all levels- school, community, district, state, national and international.

She further proposed that best practice involves:

a) listening to the voices of children
b) designing a child-centered curriculum
c) children enjoying learning
d) doing to understand – active learning

e) relating learning to the world of the child

f) teachers as learners

g) providing professional development

(p. 209)

Hence, in order to search for a good definition of good practice, we need to take into consideration voices from various parties such as students, teachers, school, community, district, state, national and international. This is because mathematics teaching is a cultural activity. Good practices in classroom teaching are shaped by all parties involved in the culture. The notion of good practice is in fact, loaded with value judgment. What is "good" to one culture might not hold true for another. With this notion in mind, I have looked into the following three sources to get a glimpse of 'good practice' of mathematics teaching as perceived by Malaysians:

1) From the Malaysian Primary and Secondary School mathematics curriculum

A brief content analysis of the latest Integrated Curriculum for Secondary School Mathematics syllabus (2004) highlighted the following emphasis in the process of teaching and learning mathematics:

a) the need to link mathematics "learning to everyday life and experiences in and out of school" (p.2);

b) the development of problem solving skills (p.3);

c) the development of logical, systematic and creative thinking together with valid reasoning (p.4); and

d) the inculcation of intrinsic values of mathematics and values of the Malaysian society which include being systematic, accurate, diligent, confident, not wasteful, moderate and cooperative, all of which contribute towards becoming a responsible citizen. (p.4)

Though not explicitly spelled out, the above emphasis may be considered as prescribed good practices which were planned or intended by the Malaysian mathematics curriculum.

Besides problem solving skills and logical reasoning, in the curriculum specification of the Integrated Curriculum for Secondary School Additional Mathematics (2004), the following were focused as the main elements in the teaching and learning of Additional Mathematics:

e) communication through mathematics, that "will develop students' ability in interpreting certain matters into mathematics models and vice versa" (p.6).

f) mathematical connections between different mathematical related topics, as well as with other learning areas (p.6).

g) the use of technology including both hardware such as computers & calculator and software related to education, websites and learning packages that are available and can enhance "students' understanding of certain concepts, providing visual representation and making complex calculation easier" (p.6).

However, to what extent are these intended 'good practice' implemented in the actual classroom? Are they pragmatic or are they too idealistic?

2) From the local research literature

The second source that I looked into was the local research literature pertaining to mathematics education. After a search of the local conference proceedings, journals, articles and unpublished paper presentations, I found only a couple of research studies related to good practice of mathematics teaching and learning.

The first one was a report of a workshop on the 'Thinking in Science and Mathematics (TISM) Project' published by SEAMEO-RECSAM (1990). The seven teacher researchers from a local school were asked to list down their perceptions of good teaching and good learning. Table 1 displays a summary of both lists.

Table 1: Perception of teacher researchers on good teaching and learning

Perception of good teaching	Perception of good learning
1. Achievement of objectives in that goals of lessons were attained.	1. That the students are happy at the end of the lesson and that they want more of the lesson concerned.
2. Good delivery and presentation	2. That the students participate by asking more questions and are thinking hard.
3. Good planning of student activities	3. That the students understand what is being taught and can apply what they have learned to solve problems.
4. Keep students busy	4. Good learning results in ability to apply knowledge to new situation.
5. Involving participation of students	5. A good learner is not satisfied with what he knows at the moment. He constantly questions the 'truth'

	of what the teacher says. He also tries to relate what he learns to his previous knowledge.
6. Involving positive changes in students	
7. Resulting in long period of retaining the knowledge acquired.	
8. Resulting in students' ability to understand, analyze, and internalize new knowledge.	
9. Resulting in change of behaviors, attitudes towards learning.	
10. Resulting in good learning on the part of students.	

Source: SEOMEO-RECSAM (1990, page 6-7).

The second was a journal article by Mohd Majid Konting (1997) which uses a totally different method of data collection. He observed 58 lessons of 16 effective mathematics teachers from one district. His assumption was that these teachers were nominated as 'effective mathematics teachers' by those in authority (including the principal, assistant principal and head of department), thus their classroom practice might reflect 'good practice'. Findings of his study show that:

> *The effective mathematics teachers were inclined to use traditional whole-class teaching strategies and to dominate classroom interaction. There was little group work and little evidence of pupil-centredness. Their actions were associated with high levels of on-task activity and good pupil behaviour. But the differences are not, to some extent, parallel with the KBSM's recommended pedagogies of pupil-centredness.*
>
> (p. 17)

Hence, there seems a mismatch between good practice as intended by the Malaysian curriculum and that were practiced in the mathematics classroom. How do we operationalize the intended good practices in the mathematics classroom then? Is it not possible to do so? What are the possible hindrances or barriers?

3) From the practicing mathematics teachers

In the year 2004-2005, my PhD student and I have carried out a Lesson Study project in two secondary schools in one district in Malaysia (see Lim, White & Chiew, 2005 and Chiew & Lim, 2005 for more details). Both project schools received the Lesson Study model of teacher professional development positively, though one of the schools shows keener interest in implementing the project than the other.

From the interviews, group discussion and the resulted lesson plan, we observed that these teachers were using three guiding principles in planning and teaching their lessons:

a) student-based activities - attempt to involve students in group or individual activities that will help them to develop or construct mathematical concepts;

b) relating mathematical concepts with real life situations by giving related examples in and out of school experience that encourage students to make connections in mathematics and make learning mathematics meaningful to students;

c) the ultimate goal of teaching is to ensure that all students can achieve good results in public examinations - drill and practice involving pass year examination questions is encouraged.

Indirectly, these principles might be termed as the main characteristics of good practice of mathematics teaching as viewed by these practicing mathematics teachers.

An exemplar lesson plan

The two case studies of Lesson Study yielded five lesson plans. One of them was on the topic: "rotation". Appendix 1 shows the complete lesson plan and the worksheet given.

Set induction (10 minutes):

To introduce the concept of rotation, the teacher asked students to describe examples of daily life situations that involve rotation. Two possible situations are the rotation of the ceiling fan or the rotation of the hand of a clock. This encouraged students to see mathematics connections with real life situations.

Teaching step 1: Developing the concept of rotation (15 minutes):

Teacher used a teaching aid (made of manila card and transparency) to elaborate the meaning of angle, direction and center of transformation as a result of rotation.

Teaching step 2: determining the image of an object under a rotation (35 minutes)

Teacher then guided the students(working in pairs with a learning aid) to determine the image of an object under various transformation of rotation such as 90 degrees clockwise and anticlockwise, as well as other angle such as 45 degrees clockwise or 240 degrees anticlockwise etc. The learning aid is made of a compass attached to a piece of tracing paper. Students solved problems in worksheet 1. Later the teacher asked students to present their answer in front and students checked their answers.

Teaching step 3: Describing a rotation(10 minutes)

The teacher distributed worksheet 2 that required students to describe a given rotation. Students work individually.

Teaching step 4: whole class discussion (8 minutes)

Students gave their answers and teacher checked orally.

Closing (2 minutes)

Teacher guided the students to summarize today's lesson. Teacher gave homework based on exercises given in the textbook.

Characteristics of Good Practices in mathematics teaching

The above lesson plan on rotation was planned and revised several times by the group of school teachers. After observing and reflecting on the lesson, the teachers were rather happy with the lesson. Ideally, they would like to have every lesson planned this way because it fulfills many of the characteristics of a good lesson or good practices. These characteristics include:

a) student centered activities that encourage conceptual understanding
b) related to students' daily life experiences
c) that the students understand what is being taught and can apply what they have learned to solve problems
d) Good planning of student activities
e) Active participation of students in fun and meaningful activities
f) Use of teaching aids that enhance student understanding

However, in practice, these teachers espoused that it was rather impossible or too challenging for them to carry out this kind of lesson everyday. This is because they met with a number of issues and constraints that do not encourage or support this kind of practices in real teaching situation.

Issues and constraints faced by the mathematics teachers

a) time constraint

Both project schools reported time as the major constraint. This can be seen from two aspects: the students' and the teachers' time constraint. From the part of the students, doing group or individual work in the classroom could be time consuming. There are fixed amount of syllabus to be covered within a limited teaching time. Hence, many teachers opt for teacher centered approach where the teacher instructs while the students listen. Besides this, from the part of teacher, student based activities demand teachers to spend much time in planning and searching for resources/ideas. Very often, teachers do not have sufficient time to plan their lesson because they are tight down by heavy workload. Again, teacher centered approach is preferred as it requires lesser preparation time.

b) Teacher's beliefs

Many teachers tend to believe that giving clear explanation with suitable examples (teacher-centered approach) is practical and sufficient to achieve most teaching objectives. It is always too time consuming to allow students to construct their knowledge through student-based activities. Furthermore, they are not confident whether their students have acquired enough knowledge and skills if the students were allowed to explore by themselves. Hence, the teachers tend to feel more certain if they can control the teaching and learning pace of their students.

c) Examination oriented culture

Examination oriented culture is prevalent in Malaysian society. Examination results especially the public examination results is used as a yard stick or accountability of school performance. From the education minister to the students' parents, everyone is very anxious about their children's examination performance. Hence, it is common for school principals to use students' performance in examinations as a yard stick to evaluate teacher's teaching competency. Consequently, this has strengthened many teachers' belief that their teaching priority is to ensure that their students pass and achieve good results in the examinations. The main duty of the teachers is to make sure that they have taught the whole syllabus before the examination.

d) Common belief of 'practice make perfect'

As our study on culture of mathematics teaching and learning in some Malaysian schools (see Lim, Fatimah and Tan, 2003) has shown that many mathematics teachers and parents, especially those from the Chinese schools tend to hold strong belief of "practice make perfect" as a way of learning mathematics. Consequently, students are usually given large amount of home work and pass year examinations questions to practice their mathematics skills.

Although teachers are exposed to student centered learning, contextual and cooperative learning approaches, they seem difficult to change the culture of mathematics teaching and learning in schools. For any new approaches that they employed, they have to meet the demands of the school principals and parents. It is thus not easy to change the culture of teaching and learning in schools.

Lesson Study as an innovative teacher professional development programme

Despite the above challenges and constraints, all the Lesson Study research project participants expressed positive feedback. From the group and individual interviews conducted at the end of the research, the participants listed the strengths of the Lesson Study process as follows:

a) Through group discussions and observing other teachers teach, they gained and enhanced both their mathematics content knowledge as well as pedagogical knowledge.

b) Upon self reflection and advice from colleagues who observed their teaching, the participants were able to rectify their own teaching errors. Novice teachers, especially have the opportunity to improve themselves by observing and learning from the experienced colleagues the skills and techniques in teaching various concepts of mathematics.

c) Lesson Study promotes a collaborative culture that enhances the professional collegial bonds within their mathematics staff.

d) Lesson study is a valuable professional development program. It was observed that participants have regarded the Lesson Study sessions as the venue to solve their teaching problems, and to develop their professional knowledge of mathematics teaching and learning.

Nevertheless, based upon our reflection on the research projects, we recommend that the following approach should be taken to ensure the effectiveness of the Lesson Study process and to be practicable in Malaysian context:

i) The Lesson Study program be monitored and supervised by the Expert Teacher ("Guru Pakar") of Mathematics, and supported by the school administrator.

ii) The Lesson Study group be made up of smaller group (3-4 mathematics teachers) to allow greater flexibility of time; group according to grade level (such as lower secondary) to reduce the constraint of time, teachers' specialization and logistic.

iii) A network of mathematics teachers be created within the district to share, learn and collaborate within the context of Lesson Study.

Conclusion

In this paper, I attempted to give a description of good practice of mathematics teaching as perceived by the Malaysian perspective. I have looked at it from the school curriculum, local related research studies and the views of the practicing mathematics teachers. There is yet to have a consensus on a definition of good practice. What I can offer is just the characteristics of good practices. This is because it is not easy to maintain these good practices. Mathematics teachers are often faced with many constraints and challenges. The main constraint being the lack of time and the examination oriented culture.

However, our case study on using Lesson Study process to promote teacher professional development seems to yield encouraging outcomes. As a result of the Lesson Study process, teachers reported a change in the nature of the staff-room discourse with a greater focus upon the Study Lessons and alternative teaching strategies coupled with a lot more sharing of ideas. Teachers are able to prepare better student based activities as they share and collaborate in preparing a lesson plan. Furthermore, as a result of the change in staff room discourse, they felt more self confident and greater support from their colleagues. Thus, perhaps the Lesson Study process which provides a meaningful context for non-threatening lesson observation, and promote collaboration and sharing within the mathematics teachers might be a possible solution or measure to enhance good practice of mathematics teaching in schools.

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Appendix 1: Lesson Plan on " Rotation"

Topic : Transformations (I) (Form 2 – Chapter 11)

Learning Area : Rotation

Learning Objective : To understand the concept of rotation

Learning Outcomes : Students will be able to:

i) identify rotation as a form of transformation
ii) determine the image of an object under a rotation; given the centre, angle and direction of the rotation
iii) describe the rotation; given the object and image of the transformation of a rotation
iv) elaborate the properties of the rotation

Duration : 80 minutes

Resources : teaching kit of transformations, clock, fan, manila cards, tracing paper, compass, worksheets.

Teaching and Learning Activities

Set Induction (10 minutes)

Teacher asks students to describe/give examples of daily-life situation that involve rotation or 'putaran' in Malay language. The teacher facilitates and guides the students to conceptualize 'rotation' with examples in their daily life.

Teacher presents the following situations (if necessary):

Situation 1: Rotation of the ceiling fan

Situation 2: Rotation of the hands of a clock

Step 1: Concept of rotation (15 minutes)

Using a teaching aid (made of manila card), the teacher elaborates and emphasizes the angle, direction and centre of the transformation of rotation.

Teacher facilitates and guides the students (working in pairs with a learning aid) to determine the image of an object under various transformation of rotation such as: 90^o clockwise and anticlockwise, 180^o clockwise and anticlockwise,
270^o clockwise and anticlockwise, 360^o clockwise and anticlockwise.
To conceptualize the transformation of rotation, teacher facilitates students with examples such as 60^o clockwise, 45^o anticlockwise, 250^o clockwise etc.

Teacher uses the teaching kit provided and demonstrates it on the board with a few examples. Teacher guides the students to conceptualize the properties of the rotation.

Step 2: Determining the image of an object under a rotation (35 minutes)

Teacher distributes worksheet 1 that requires students to determine the image of an object under a rotation. Teacher demonstrates (using the teaching kit) to the students how to use the learning aids (tracing paper and compass) to solve the problems in worksheet 1. Students work in pair and the teacher checks students' answers.

Students present their answers on the manila cards (the teacher prepares the manila cards). Using the manila card, the students determine the image; given the transformation and object by the teacher. Students check their answers.

Step 3: Describing a rotation (10 minutes)

Teacher distributes worksheet 2 that requires students to describe/elaborate the rotation; given the object and image of the transformation of rotation. Students work individually in worksheet 2.

Step 4: Discussion (8 minutes)

Students give their answers in worksheet 2 and the teacher checks the answers orally.

* Note: There are two answers for every question and teacher should sought both answers from the students. Using the transformation kit, teacher demonstrates and explains why there would be two answers for every transformation of rotation (if necessary).

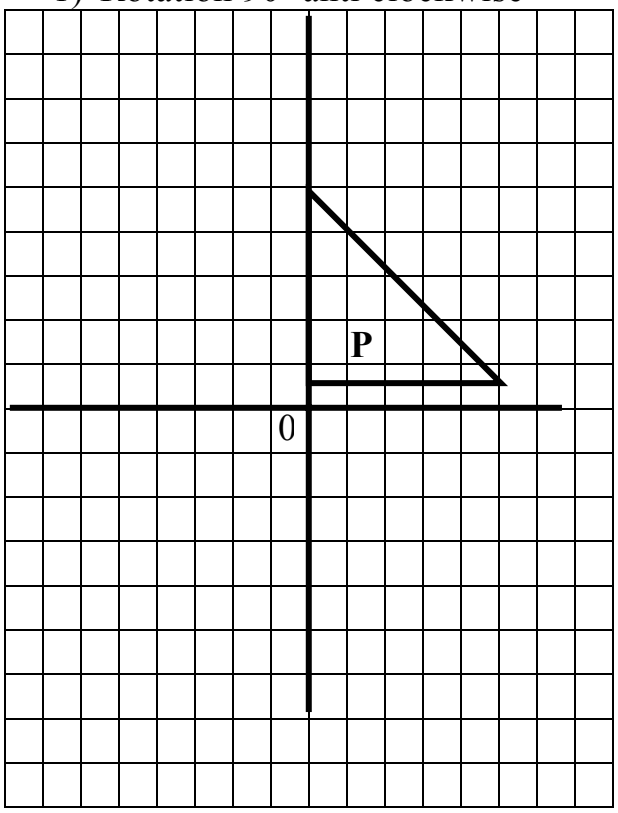
Closing (2 minutes)

Teacher guides the students to summarize today's lesson.
Teacher gives homework from the textbook: pg. 96, exercise 11.4, question 6.

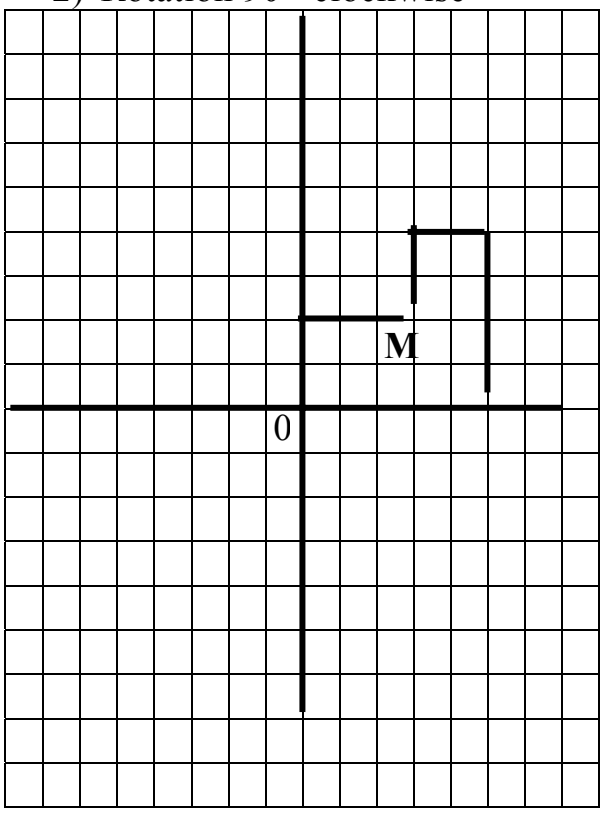
Worksheet 1

Instruction: Draw the image of the object under the rotation stated about the origin.

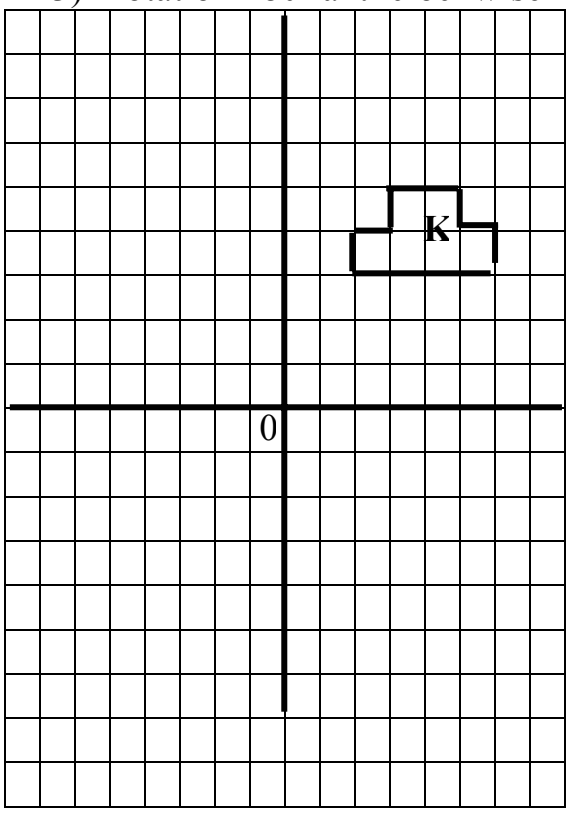
1) Rotation 90^o anti clockwise



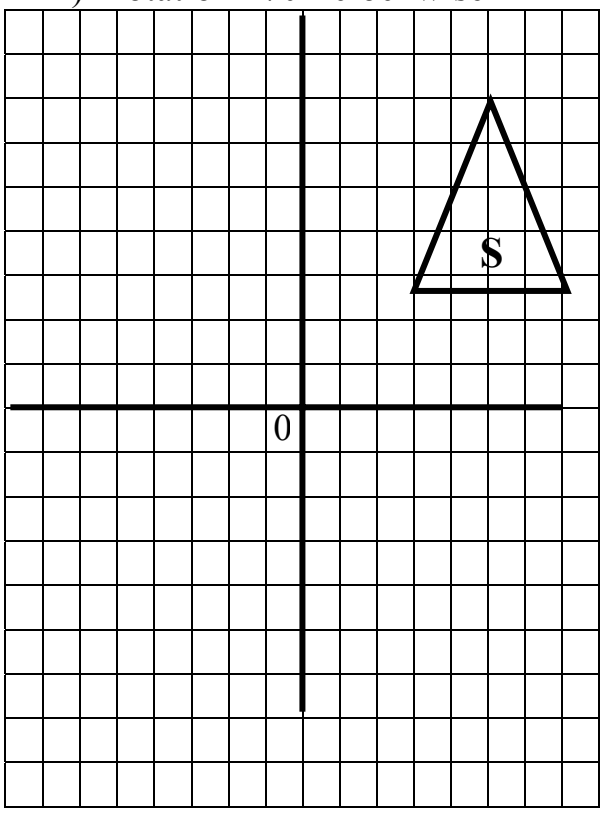
2) Rotation 90^o clockwise



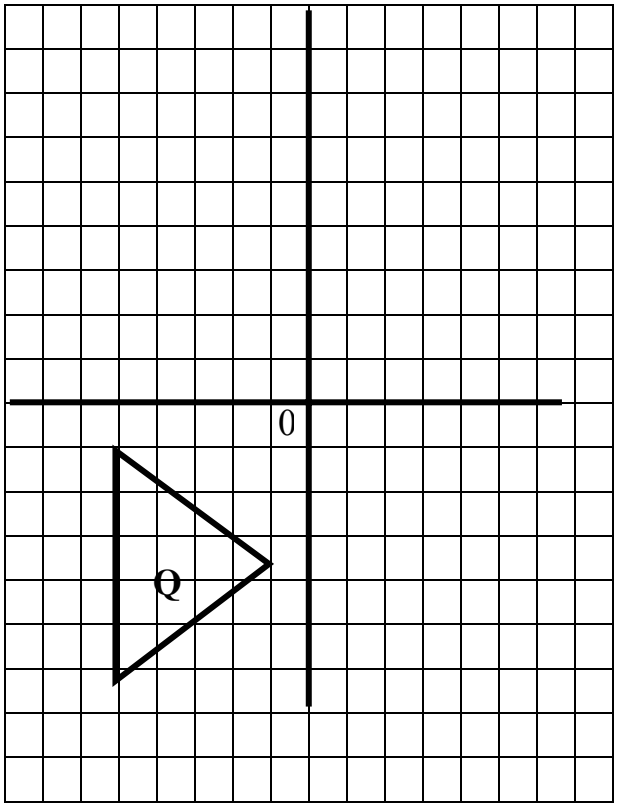
3) Rotation 180^o anti clockwise



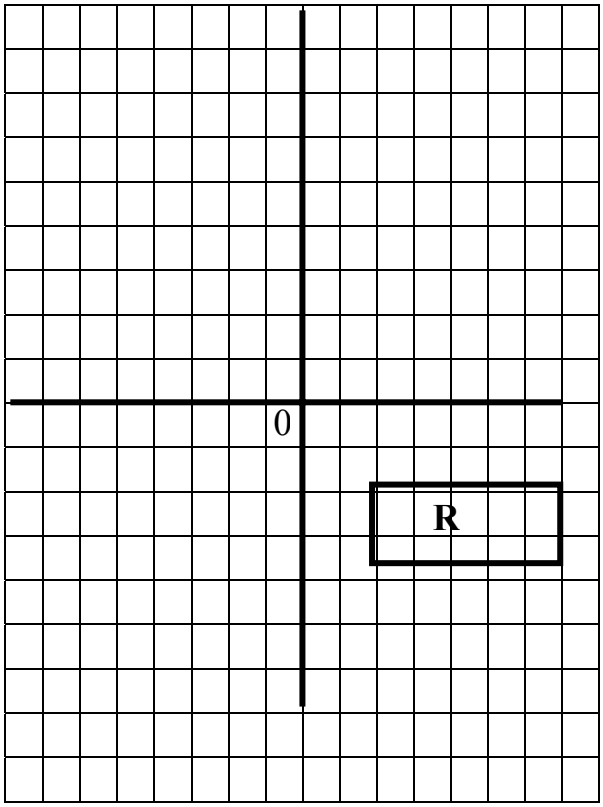
4) Rotation 270^o clockwise



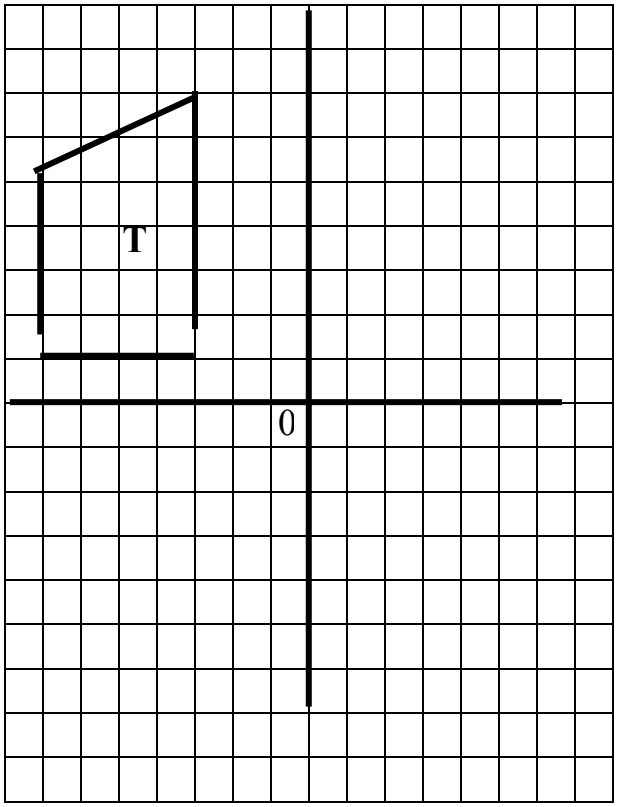
5) Rotation 90^o anti clockwise



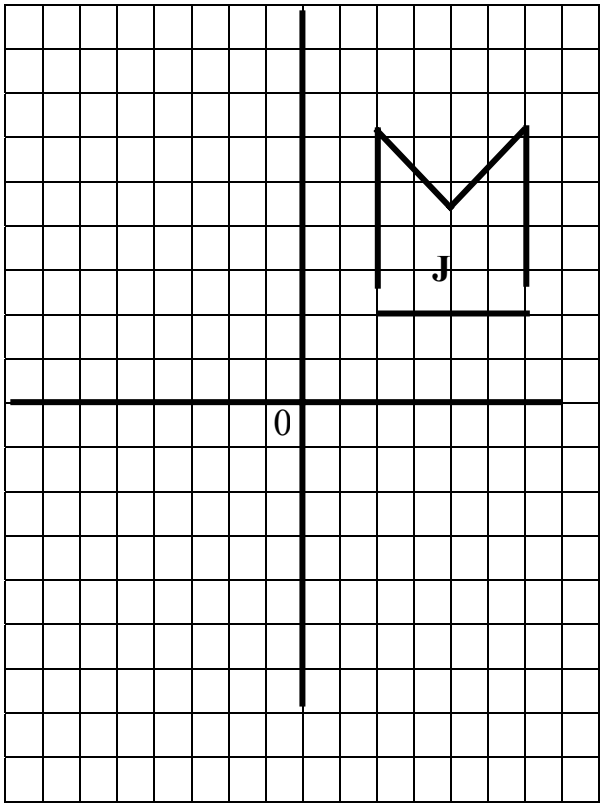
6) Rotation 90^o clockwise



7) Rotation 180^o anti clockwise



8) Rotation 270^o clockwise



PURSUING GOOD PRACTICE OF SECONDARY MATHEMATICS EDUCATION THROUGH LESSON STUDIES IN INDONESIA

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Starting in 1999 and lasting in 2005, the extending of IMSTEP_JICA Project resulting the piloting activities through Lesson Studies for searching good practice of secondary mathematics teaching in three cluster site West Java, Central Java and East Java. Results of the studies significantly indicated that there are improvements of the practice of secondary mathematics teaching learning processes in term of teaching methodology, teacher competencies, students achievements, alternative evaluation, teaching learning resources and syllabus. The results of the project support the government efforts in improving secondary education by introducing the new curriculum. However, there are still great challenges for both educationists and practitioners to establish good practice of secondary mathematics teaching. One of the proposed solutions is to en-culture lesson studies activities by e.g. learning them from Japan contexts.

Key Word: good practice, lesson study, secondary mathematics teaching learning

OVERVIEW

Some philosophical backgrounds of the nature of good practice of teaching need to be discussed as the references of the efforts to put on the grounds for the long-term orientation of improving mathematics education. Those notions cover the questions to mean education whether: (a) as the investment or as the need for society?, (b) as the obligation or as the awareness of the students?, (c) as the competition or as the collaboration?, (d) as the product or as the process, etc. Although the examples of good practice differ in some contexts, there are some features common to the way the children were working such as in/formality of typifies classroom, clearness of the objectives, ethos of the school, flexibility and variety of teaching styles. The quality of teaching was the strongest feature common to all the examples of good practice; while the adequacy of good practice may outlined on the basis of the context of the ideal practices.

Brown in Riley (1992) suggested that the features of good practice include fostering a positive attitude to mathematics; emphasis on the application of mathematics; well-planned work; children formulating testing and revising hypotheses; work and pattern

and relationship; a variety of approaches to calculation used; sensible use of the calculator; extensive experience of measurement and estimation; clear policies on mathematics; individual, group and whole class work as appropriate; opportunities for co-operative work; positive and well timed teacher intervention; meeting needs through the differentiation of work; use of practical and first-hand experience; cross-curricular work; appropriate reflection of cultural diversity; relevant exploratory work; stimulating working environment; effective teaching.

The Third International Mathematics and Science Study (TIMSS), from the videotaping study of classroom instruction of mathematics lessons in the U.S., Germany, and Japan, found that good practice of mathematics teachings were more likely to target mathematical thinking and seeking to teach students how to solve a particular kind of problem or carry out a specific procedure. Further, it suggested that the concepts of mathematics were far more likely to have been developed rather than simply presented as rules. It included multiple ways to solve mathematical problems and asked students to perform tasks that were not "routine." TIMSS suggested that good practice of mathematics teaching should encourage the teachers to help students make explicit connections between parts of the lesson to previous knowledge, and/or to statements and problems from earlier parts of the lesson. Good teaching practice should be supported by the research on how children learn and, particularly, on how they learn mathematics.

While in Japanese context, on which Lesson Study has its culturally deep rooted, good practice of mathematics teaching (Masami ISODA) may perceived as it is visible, recordable in the classroom and can be showed to other people. Further, it may be known as a good approach in an economy in which there is a teacher who is well known by its approach. Accordingly, good practice of mathematics teaching should be useful for the reform of mathematics education as a whole and many teachers may have their wish to do the same approach. Good practice of mathematics teaching also should encourage the improvement of teacher training program. According to him, the challenges for the practitioners who develop good practice consist of capacity to describe the nature of good practice? Why we can say it as good practices? What kind of reform is expected by such kinds of practices? What kind of the setting in curriculum standard for explaining why it is good. And what kind of relation of good practice to the world of mathematics education research movement?

One significantly reference for Lesson Study performed by Education Development Center, Inc. ("EDC") of Massachusetts. It organized divisions and centers of various sizes of Lesson Studies and develops Lesson Study Communities Project to carry research to introduce teachers to lesson study, to build a community of teachers interested in lesson study, to enhance teacher knowledge of mathematics and pedagogy, and to learn how the Japanese lesson study model can be adapted to become a successful professional development model for U.S. secondary school mathematics teachers. Indonesian educationist as well as practitioners may be able to

learn from Japan and other country to perform good practice of mathematics teaching in secondary schools.

OBSERVABLE GOOD PRACTICE AND IDEAS VALUES BELIEFS OF MATHEMATICS TEACHING LEARNING IN INDONESIA

The efforts of pursuing good practice of mathematics teaching learning in Indonesia, starting from 1994 up to present, have its ideal values beliefs, political as well as empirical and pragmatic ground. In the conceptual framework, it can be seen that observable good practice components can be directly related with educational questions 'What should children learn ?' and 'How should children learn and teachers teach ?'; and ideas values beliefs can be directly related with the questions 'Why should children be educated in this way ?' and 'What is an educated person ?'. As they are elaborated from socio-constructivists approach, that teacher is not only to implement the curriculum but also to develop it. Teaching and learning in the classroom should not always be directed teaching in which teacher dominated activities and initiations; however, teacher needs to accommodate students' initiatives and students' needs. Therefore, teacher needs to implement flexible method of teaching, in which students' performance and achievements can be assessed during the processes of learning activities. It implied that teaching learning process will be student centered rather than teacher centered, in such away that students have various experiences and opportunities to consciously uncover the nature of what they are learning.

Mixing from values beliefs and empirical evidences, there are currently demands in Indonesia, that any educational reform should handle the issues of (a) how to promote interactive curriculum rather than instrumental curriculum?, (b) how to promote student centered approach rather than teacher centered approach?, (c) how to promote students' initiation rather than teacher's domination?, and (d) how to promote simple and flexible curriculum rather than crowded and tight-structured curriculum? While in term of observable good practice, there were demands that teachers need to have a chance to reflect their teaching in such away that they may move from older paradigm of teaching to the new one. Teachers may move from emphasizing the "teaching" to emphasizing the "learning"; they may move from the act of "transferring teacher's knowledge" to "constructing students' knowledge".

Figure: Most Indonesian teachers implement Traditional Teaching (2001)

There are also demands that teachers may move from "instructing" to "serving"; they may move from "product oriented" to "processes oriented"; they may move from "classical teaching" to "individual teaching"; they may move from "single method of teaching" to "various method of teaching"; and they may move from "theoretical" to "hands on activities". To develop the aspects of students' life-skill teachers need to have a lot of time, passionate, and extra-efforts in such away that teachers are able to motivate their students, to give them the chance to develop their skills and experiences. In the socio-constructivist approach, teachers are expected to fully concentrated and focused at their teachings and its preparations. It made them to be able to promote their creativities in developing their various method of teaching such as discussion, investigation, laboratory practice and demonstration.

The currently studies on mathematics teaching practices in Indonesia indicated that, under the implementation of the Curriculum 1994, student's process skill and student's achievement are still low; contents on Mathematics were crowded; too many time consuming administration stipulation for teachers; there were mismatch among the objectives education, curriculum, and National Evaluation system. Further, National Leaving Examination assessed the children's ability cognitively only and considered individual differences inappropriately; while University Entrance Examination system was considered to trigger school teachers apply much on goal oriented rather than process oriented. Observable practices of mathematics teaching in the period of the year 2001-2003 also indicated that many teachers still have difficulty in elaborating the syllabus; a number of mathematics topics are considered to be difficult for teachers to teach; a significant number of children consider some mathematics topics as difficult to understand; teachers consider that they still need guidelines for conducting teaching process by using science process skills approach.

PURSUING GOOD PRACTICE OF SECONDARY MATHEMATICS TEACHING THROUGH MEDIUM-SCALE OF LESSON STUDIES

In the fiscal year 2001-2003, a medium scale of piloting of Teaching Learning Model of secondary mathematics and sciences through Lesson Study has been carried out by

IMSTEP-JICA in collaboration with UPI Bandung, UNY Yogyakarta, and UM Malang, in which Japan Government supported facilities, training as well as Educational Experts.

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Method

The piloting activities were carried out in three clusters i.e. West Java (Bandung), Central Java (Yogyakarta), and East Java (Malang). Following are the sites:

	West Java (Bandung)		Central Java (Yogyakarta		East Java (Malang)	
	Year 2001/2002	Year 2002/2003	Year 2001/2002	Year 2002/2003	Year 2001/2002	Year 2002/2003
Classes (schools)	1	2	1	3	1	1
Teacher involved	1	2	3	3	1	1
Lecture involved	4	4	4	3	4	4

Table: Three cluster sites of Lesson Studies of Mathematics Teaching (IMSTEP-JICA Project)

The Lesson Studies were developed in which the teachers, in collaboration with Lecturers and Japanese Experts, tried out some teaching models at schools. The Lecturers of Teacher Training Program and School Teachers worked collaboratively, composes some numbers of Lesson Studies. The grounds of the Lesson Study activities were reflecting and promoting the new paradigm of the secondary mathematics and science education, in which learning activities are not only perceived pragmatically and short-time oriented but also to be perceived as a long-life time purposes.

The objectives of those Lesson Study activities were to contribute the improvement of secondary mathematics education by pursuing good practice of mathematics teaching. Lesson Studies for secondary mathematics were carried out by mainly Classroom Action Research approach. They carried out to improve the teaching learning practices and to find more appropriate methods for facilitating students learning. Teachers' experiences have been shared with other teachers and the lectures. The specific objectives of Lesson Study activities are: (1) to develop instrument and equipment for teaching learning process, (2) to develop teaching method and model for teaching learning process, (3) to develop teaching material for teaching learning process, and (4) to develop teaching evaluation for teaching learning process.

Lesson Study activities let the teachers to reflect and evaluate, in cooperation with lectures or other teachers, their paradigm of teaching. Approaches of Lesson Studies covered (a) students cooperation with others in their learning, (b) contextual teaching and learning, (c) life-skill, (d) hands-on activities, (e) interactive process oriented

curriculum and syllabi development, and (f) teachers and students autonomous. From those three sites of study, there can be produced the notions of educational improvement, in term of teacher, student and lecture.

Result

The results of Lesson Study could be inferred from the view of students, teachers, and of lecturers. Evidences were collected through observations, questionnaires and interviews. It can also be noted the strengths and weaknesses of the activities. Those are as the following:

Teacher

1. The teachers felt that the class is getting more alive. However, all the above progresses have to be paid by spending longer time for preparation.
2. The teachers have to spend longer time for preparation, have sufficient skill to run the experiments and use various tools.
3. Lesson Study gave positive results because it could improve teachers' professionalism in finding variations of teaching approaches, and teaching methods.
4. It could also improve teachers' skills in classroom management and in questioning and in developing creative ideas.
5. Through Lesson Study, many teachers were introduced some innovations in mathematics and science teaching and learning.
6. The new model was introduced to teachers to increase the variation of alternatives on how to conduct classroom teaching and learning process.
7. Teachers stated that now, they have more choice to teach certain units of studies.
8. Teachers involved in these Lesson Study activities developed their competencies in teaching mathematics and science.
9. Competencies developed for teachers are realistic approaches (RMA), authentic assessment, and constructivist approach.
10. Teachers involved in these Lesson Study activities felt that they have to think and develop new ways on how to let students learn and construct their own concepts.
11. They expressed their impressions that their creativity was improved.
12. There were indication that teachers' skill to communicate, to deliver questions, to carryout discussion method was improved e.g. in order to stimulate students to think, teachers asked questions and by doing these, questioning skills were improved.
13. There were also evidences that teachers' perceptions of their students' learning in Lesson Study activities were positive.
14. Teachers stated that Lesson Study activities needed to be continued or to be extended in order that they could continue to develop mathematics and or science teaching.
15. Teachers stated that Lesson Study activities were useful to support the implementation of competence-based curriculum.

Student

1. Good responds were heard during the meeting, especially the students' enthusiasm, students' involvement in doing experiment and discussion.

2. The students respond to the contextual teaching and learning approach during Lesson Study period may be indicated by their participation in experimentation, discussion, and presentation.
3. More than 80% of the number of respondents were actively involved in the experimentation, approximately 75% involved in discussion, and more than 75% involved in presentation.
4. It can be concluded that nearly 3 out of 4 students were actively involved in the contextual teaching and learning process.
5. Nearly 70% students were happy with the teaching learning process using a lesson plan.
6. More than 57% of them were happy with the lesson plan and the involvement of teacher during teaching and learning process is still high, more than 95%.
7. Students could develop responsibilities in their learning and be more active in finding learning resources out of the classroom.
8. Students get more skillful in sharing ideas and communicating their mathematics activities
9. Innovations practice tried out at Lesson Study schools could also improve students' enthusiasm, motivation, activities, and performance.
10. The innovation in approaches and media of learning bring good results for students. Most students in each class were enthusiastic in learning using the new media, methods, or approaches.
11. It was indicated that most of students expressed their happiness during the Lesson Study activities; the reason were: (a) the lesson was not so formal, (b) the contents were easier to learn, (c) they were able to express their ideas, (d) they got much time to discuss with their classmates.
12. The reasons of improving students' motivation were: (a) they were able to detect their competencies any time, (b) contextual teachings made them understand the usage of learning certain subject-matter, (c) realistic teaching made them not be boring.
13. There were more activities done by students in science laboratories, especially activities to improve their process skills.
14. Most of the students stated that they liked to learn with hands-on activities, discussion, demonstrations, teaching aids and worksheets.

Lecturer

1. Lecturers got experience in developing teaching materials for schools and know more about problems faced by teachers.
2. They could also develop more creative ideas to find better methods of instruction with teachers and better hands-on activities by utilizing local materials.
3. Through Lesson Study activities, lecturers were also benefited in knowing more about the problems faced by teachers and schools in conducting mathematics and science teaching and learning.
4. Lecturers' experiences were improved due to the fact that in Lesson Study activities, they in cooperation with teachers, should develop teaching guide, teaching materials and assessment methods.

Constraint

1. Those are the tendency of teachers to keep on doing "teacher centered activities" instead of trying new ways of teaching which need thought, energy and time to develop, to plan, and to implement.
2. These resistances for change are caused by the "crowded curriculum" that have to be finished and the fact that there were too many students in a classroom.
3. Lecturers and teachers should also need time to introduce and learn new innovations. They should be very patient in developing new things; they could only develop one thing at a time.
4. Based on their experience in conducting processes, there were different perceptions of those lecturers and teachers on new paradigm.
5. Some teachers tend to skill use the "old" orientation in teaching and learning processes, which is to try to achieve the highest possible score in final examination.
6. In order to do that they usually practice "teacher centered" and "product oriented" teaching and learning processes and forget to develop students' thinking processes skills.
7. Based on the implementation of Lesson Study activities, it was found that some aspects of Lesson Study activities have difficulties to implement in term of teachers, students, class management, equipment or facilities, and schooling system.
8. Number of students in each Lesson Study class was relatively high i.e. 35 or more; this made the teachers have difficulties to facilitate their needs in learning activities. With a large number of students in the class, it was difficult for the teachers to monitor and supervise their activities.
9. Due to the fact that we do not implement moving class, it needs more time for the teachers to prepare equipments and other facilities.
10. Another difficulties faced by the teachers related to the development of students' worksheets was that teachers have some problems on how to prepare many kinds of worksheets, to decide the topics to be piloted and to prepare teaching guides.

Figure: Group discussion in piloted teaching practice (2003)

DISCUSSION AND CONCLUSION

There were strong evidences that Lesson Studies activities improved students' enthusiasm, motivation, activities, and performance. It also improved teachers' professionalism in terms of teaching performance, variation of teaching methods/approaches, collaboration. Lecturers got knowing more about the problems

faced by teachers. It was take time for teachers to shift from teacher-centered to student-centered. Teachers developed teaching methods based upon more hands-on activities and daily life utilizing local materials. Students were active learning and involved in discussion to share ideas among classmates. Students enjoyed learning science and math during Lesson Study activity due to some reasons. According to students' respond, the lesson was not so formal, the contents were easier to learn, students able to express their ideas, students got much time for discussion with their classmates, more experiment science and math. Teachers got alternative method to let students learn and construct their own concepts. However, teachers took time to get used to develop teaching model by their own.

The Lesson Study project was proven to be very effective in lifting students' enthuciastic in learning science, helping students to develop their experimental and discussion skill, giving opportunities to students in developing their own scientific concept by themselves. It was also reported that by using constructivism approach, the students may find out their best style of learning. Competition rises among groups of students in presenting the results of their work and defending their presentations. This forces students to learn theory more on their own. As a result of Lesson Study activities there were many teaching material developed either by lecturers and teaching together or by lecturers or teachers themselves. Those materials were either developed by lecturers or teachers in their own classroom or by lecturers and teachers together during Lesson Study activities. In general lecturers and/or teachers developed the teaching materials after thinking extensively what and how to develop teaching materials for a certain topic, and then develop the materials. Further they try out the teaching materials in their classroom and revise those according to the result of the try out.

The results of Lesson Study activities and exchange experiences come to a suggestion that to improve mathematics and science teaching in Indonesia; it needs to deliver obvious messages to the government, teachers and head-teachers or schools. Learning from study, it was also suggested that to promote good practice of mathematics and sciences teaching, the teachers need to en-culture their efforts in inovating teaching learning processes which meet to academic students needs, encouraging students to be active learners, developing various strategic of teaching, developing various teaching materials, and in developing teaching evaluation. In developing teaching learning methods, the teachers need to: plan the scenario of teaching, plan students activities, plan teachers' roles, distribute the assignments, develop assesment methods, and monitor the progress of students achievements.

To develop their experiences, the teachers also need to participate frequently in such kinds of workshops or seminars. By using those teaching materials teachers could conduct the teaching and learning process more efficiently. Students enjoyed their learning process because they were involved in observing and doing things. Those teaching materials also improve students' motivation and interest in learning the materials. Although there were may kinds of teaching materials that have been

developed through those Lesson Study activities, there still more topics that need to have or to have better teaching materials. Therefore lecturers from three universities need to have further collaborative work to develop more teaching materials in the future.

The study also recommended that to encourage educational innovations, the head-teachers need: (1) to make good atmosphere for teaching and learning, (2) to promote to implement various teaching methods and teaching learning resources, (3) to give the chances for the teachers and their students to perform their initiatives, (4) to promote cooperative learning, (5) to promote research class as a model for educational innovations (as Japanese teachers do), (6) to support the teachers to be the developer/maker of the curriculum, (7) to promote teachers' autonomy in developing model of teaching learning activities, (8) to implement school-based management, (9) to encourage students' parents participations, and (10) to promote cooperation with other educational institutions.

Further, the study also recommended that to improve the quality of mathematics and sciences education, the central government needs to: (1) implement more suitable curriculum i.e. more simple and flexible one, (2) redefine the role of the teachers i.e. teachers should facilitate students' need to learn, (3) redefine of the role of principals; principals should support the professional development of teachers by allowing them to attend and participate in scientific, meetings and trainings, (4) redefine the role of schools; schools should promote school-based management, (5) redefine the role of supervisor; the supervisors need to have similar background with the teachers they supervise in order to be able to do academic supervision, (6) improve teachers' autonomy to innovate mathematics and science teaching and learning, and (7) promote better collaboration between school and university; communication among lecturers and teachers should be improved; these could be done through collaborative action researches and exchange experiences through seminars and workshops, (8) redefine evaluation system, and (9) to extend project for promoting new paradigms and educational innovations.

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GOOD PRACTICES IN MATHEMATICS TEACHING AND TEACHER DEVELOPMENT

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Since 1992, the Singapore mathematics curriculum has been revised several times to encourage teachers to help pupils develop competencies that are useful for the global, technological economy. This paper provides an overview of what is considered good mathematics teaching. The focus of this paper is on teacher development. How can and to what extent do teachers develop such good practices?

INTRODUCTION

In 1992, the Singapore mathematics curriculum was changed to make problem solving the focus of the curriculum. In 1997, the Singapore Ministry of Education launched an initiative Thinking Schools, Learning Nation to encourage schools to explicitly teach thinking skills. In 2001, the mathematics curriculum was revised to reflect this emphasis. As a result, textbooks included a wider variety of problem-solving heuristics. There is also a conscious effort to teach problem solving explicitly. The focus, however, is on skills and techniques. In 2003, the Singapore Ministry of Education launched a follow-up initiative Innovation and Enterprise to encourage schools to help pupils develop good habits of mind.

In 2004, the Prime Minister asked, in his national day speech, schools to teach less so that pupils can learn more. The theme Teach Less, Learn More philosophy is encouraged in schools. Teachers are encouraged to help pupils master the basics well and to apply these basics in a wide range of situations rather then attempt to tell pupils everything. The Teach Less, learn More call underlines the Singapore Ministry of Education's effort to help pupils develop thinking skills and thinking habit. In 2005, the Ministry set as its aim to nurture every pupil. In line with this, all grade one class size was reduced to thirty (previously it is typical to have forty pupils in each class) and teachers are encouraged to use different strategies to help pupils develop in an effective and engaged manner. Presently, grade one and grade two teachers are using the SEED approach in their classrooms. SEED stands for strategies for effective engagement and development.

In 2007, the mathematic curriculum is revised to emphasize the recent initiatives. Specifically, teachers are encouraged to de-emphasize paper-and-pencil computation and to emphasize mental computation and skills like visualization.

Overall, the education system and the mathematics curriculum aim to help pupils develop competencies that are useful in a global, technological knowledge-based

economy. What are some good practices that contribute to the realization of this vision? How do teachers develop such good practices?

GOOD PRACTICES

In analysing the recent initiatives by the Singapore Ministry of Education, good practices in the mathematics classroom can be characterized as the following:

- Good practices provide pupils opportunities to develop competencies and attitude that put them in good stead in the global, technological economy.
- Good practices aims to develop good thinking by enhancing pupils' thinking skills and thinking habit.
- Good practices instil among pupils a belief that they are able to extend their own knowledge.
- Good practices engage pupils in the learning process.
- Good practice is effective as in *all* pupils develop key ideas in mathematics.

In a project involving five primary schools in Singapore, one part of the study aims to study how teachers can be engaged in developing innovative approaches and exemplify good practices. Briefly, the Think-Things-Through (T^3) Project provides worksheets for teachers to use in their mathematics lessons. Teachers are encouraged to read notes for teachers provided by the research team and to discuss with each other before the implementation. The two sources for teacher development are (1) the worksheets and (2) the discussion. The worksheets are available at http://math.nie.edu.sg/T3

In traditional mathematics classrooms, teacher actions are limited to providing explanation. In good practices, teachers should be able to engage in a wider range of roles. Other than providing explanation, are teachers able to model certain way and habits of thinking? Are they able to guide pupils to think in a certain way? Are they able to provide the necessary materials and environment to create opportunities to engage in certain way of thinking? In brief, do teachers take on more roles in the learning process?

AN ILLUSTRATION

The lesson was for a grade three class. Pupils were supposed to for the letters I and T using sticks. Specifically, each letter I is formed using three sticks and the letter T using two sticks. Pupils were given 19 sticks and asked to find the number of Is and Ts that can be formed. Subsequently, the pupils played a game where they may use any of the observations that they have made in the first part of the lesson.

- Good practices aims to develop good thinking by enhancing pupils' thinking skills and thinking habit. Various parts of the lessons required pupils to make observations, make generalizations and extend their thinking.
- Good practices instil among pupils a belief that they are able to extend their own knowledge. Pupils were expected to use the ideas from the first part of the lesson to determine a winning strategy for the game.
- Good practices engage pupils in the learning process. Pupils were actively involved in solving the problem. They were given concrete materials to work with. As they were paired up, pupils were talking and discussing with each other.
- Good practice is effective as in *all* pupils develop key ideas in mathematics. The problem is accessible to every pupil. Every pupil in the class was able to achieve some degree of success in this activity. The use of sticks made the problem even more accessible. In the game, pupils were asked to observe generalization. The activity did not emphasize the computation aspects. Instead big ideas such as making generalization were given emphasis.

This lesson was designed by the teacher. Previously, she had conducted lessons prepared by the research team. The lesson that she designed and conducted had features that are similar to the lessons designed by the research team.

TEACHER-INNOVATOR MODEL

A teacher development model for good practices is proposed to investigate the extent of teacher development through activities such as lesson study. The model comprises four stages.

Level 0

Teachers at this level are indifferent to the stimulus provided. Such stimuli include the lessons designed by the research team and any discussion the teacher may have with his colleague. Other than the lessons designed by the research team, no change is detected in other lessons.

Level 1

Teachers at this level respond to the stimulus provided in a superficial manner. For example, the lessons they design are identical to those designed for them. Any modifications are superficial.

Level 2

Teachers at this level respond to the stimulus provided in a structural manner. For example, the lessons they design are structural modifications of the lessons designed for them.

Level 3

Teachers at this level have become innovators of good practices. They no longer modify lessons they receive from the research team. Instead, they design their own lessons.

Using this model, it may be possible for us to answer the questions to what extent do teachers develop such good practices. Subsequently, we may be able to study the conditions under which each level is achieved.

DEVELOPMENT OF EFFECTIVE LESSON PLAN THROUGH LESSON STUDY APPROACH: A THAI EXPERIENCE

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INTRODUCTTION

One of the main focuses of educational reform movement in Thailand call for the development of good or effective lesson plan aligned with the new 2001 curriculum. In order to respond to this, the study of the lesson plan development using Lesson Study Approach according to the Education Reform Act has been conducted by the cooperation of the Faculty of Education, Khon Kaen University, Plan Organization of Thailand and the Education Area 5 (Jurisdiction of Supervisory Area 5). The study has the objectives of developing the ability of the leading teachers in Khon Kaen province to develop good lesson plan according to the Education Reform Act 1999 which emphasizes the reform of student's learning processes (ONEC, 2001, Sintoovongse, 2002). The leading teachers are expected to have the understanding and skills in the development of lesson plans by using the Lesson Study Approach. In addition, they are required to implement the plans in the actual classrooms, to follow up on the results, to expand their practice to the other teachers in the education areas to which they belong. The project evaluation was carried out through project exhibitions and presentations.

Based on the positive outcomes, the project has proved to be valuable in teacher development. All leading teachers were interested in participating in the activities in order to fully enhance their capacity. The continuous effort in expanding the knowledge to the network teachers in the other education areas was one of the evidence showing the success of this study.

This report is a summary of the study in the lesson plan development based on the Education Reform Act 1999. Since the operation of the study has been a great impact on the northeastern region of Thailand in educational reform movement, it also includes some suggestions that would be useful for the education improvement of the other regions and organizations in the future.

WHAT IS A DESCRIPTION OF GOOD PRACTICE?

What should be called good practice in educational reform movement in Thailand has to satisfy some of the following criteria: aligned with the 1999 Education Act and the new 2001 curriculum, supporting teacher as a researcher trend, creating teacher networking in the region, in particular among schools and higher education institutions like university, teachers colleges, or vocational colleges, if it is the work of graduate school, needed to pull the theory from the shelf. This project at least satisfies most of the criteria just mentioned.

In this project, the participants are the leading teachers in the Education Area 5 of Khon Kaen province. Among 48 participants, there are 30 women and 18 men, 28 of whom teach in the primary school level and 20 of whom teach in the secondary school level.

The project aims to develop the teachers to be researchers at the same time to improve their teaching practice. It is believed that research leads to positive changes and continuous development by the aid from fellow teachers. The content of the curriculum comprises the following process of the Lesson Study Approach (Stigler and Hiebert, 1999):

1) Defining the problem

The Lesson Study Approach concentrates on the process of solving problem. To define the problem is the way to the motivation in working and building framework of the group of teachers. Problems defined may be general problems (such as how to generate students' interest in mathematics) or in-depth problem (such as how to develop the understanding in adding the unequal fraction). Normally, the most common problems teachers encounter are those caused by the teaching experience that affects students' learning or those related to the national policies.

2) Planning the lesson

After the learning goals are set by the group of teachers, they start the meeting to plan the lessons. The goal of lesson plan is not only to come up with effective lessons, but also to improve the understanding with students. The initial plan is presented to the teachers' meeting of the whole school to get feedback to improve the plan. This step may take up to a month or several months before it is ready to be used in the classroom.

3) Teaching the lesson

This is the step of bringing the lesson plan into the class. The class schedule and the instructors are set by the teacher group. The teacher who is responsible for the class must be involved in every single step of lesson planning. As the teaching starts, the rest of the participants are observing the class, taking detailed notes so that sufficient information is available for reflection of the lesson in the next step.

4) Evaluating the lesson and reflecting on its effect

After the class is over, the teachers evaluate and reflect on the lesson. The first who gives the opinions or reflects on the lesson is the teacher who teaches the class. The teacher focuses on how successful the plan is and what the problem are. Next, the other participants show their reflections only on the lesson that they planned together but not on the instructor's performance. Everyone takes responsibility of the results from the lesson. The opinions on the lesson will lead to the lesson improvement.

5) Revising the lesson

The teachers revise the lesson by using the information gained from classroom observation and the lesson reflection. The revision might involve the development of education technology, the class activities, the problems coming up in the class and the questions emerging along the process. Often, the lesson revision emerged from the students' misunderstanding of the lesson during class activities.

6) Teaching the revised lesson

The revised lesson is used in the classroom by either the same teacher or the new one. At this stage, all teachers in the school participate in the classroom observation.

7) Evaluating and Reflecting

As it comes to this stage, not only the teachers in the school but also the specialists from outside of the school are involved. Like in step four, the teacher taking control of the class is the first who evaluates the lesson. How the students learn from the lesson is the main point to be considered as the lesson reflection proceeds. Some other points to be mentioned include the lesson design by considering theories and principles underlying the design. The reflection also includes the discussion of knowledge gained from planning the lesson and bringing it to the real classroom.

8) Sharing the results

Although the Lesson Study Approach is based on a case study, the results are generalizable because Japan uses the same basic curriculum. Therefore, it is encouraged that the results be published and presented in the annual regional and national conferences.

The premises of the lesson plan development using the Lesson Study Approach are summarized below:

1) Develop continuously

2) Maintain students' effective learning as the ultimate goal

3) Focus on developing teaching practice in the real classroom

4) Focus on the process of the teachers' mutual learning

5) Perceive the teachers' roles as contributors to the development of body of knowledge and practice in the teaching profession

HOW IT IS DEVELOPED?

The procedure of operating this project has been designed based on the framework of Lesson Study Approach mentioned earlier. It comprises of 5 phases as follows:

Phase I: Developing the network of the co-operation

To develop the co-operation in the development of the lesson plan in Mathematics project due to the 1999 act of education between the Faculty of Education at Khon Kaen University , Plan Organization of Thailand and The academic area of Khon Kaen.

Phase II: Setting the workshop for improving the leading teachers

The workshop, held on April 19-23, 2004 at the Faculty of Education, Khon Kaen University, had the objectives of creating the understanding in improving the skill of making the lesson plan using Lesson Study Approach.

Phase III: Training in the class

On May 17 to Sept 24, 2004, the leading teachers brought the improved learning plan to the class. The steps of this stage are presented as follows.

1) The Follow –up

1. Lesson plan check up: The Mathematics teachers participated the workshop of designing the lesson plan using the Open Approach sent the plan back to the committee to be checked up for the suggestions and improvement.
2. Suggestions for the improvement of lesson plans: The committee gave the suggestion for improvement of the lesson plans.
3. The appointments for observation and supervision: The organizing committee directly made appointments with the teachers in the key learning area group for the follow-up of the usage of the lesson plan. The table showing the appointment for observation and supervision is shown the next.

2) The reflection of using the lesson plan

The group of teacher used the following procedure:

1. Group discussion on the implementation of the learning plans at each basic education level
2. Summary of the discussions for presentation
3. Presentation within the group
4. Analysis and group discussion

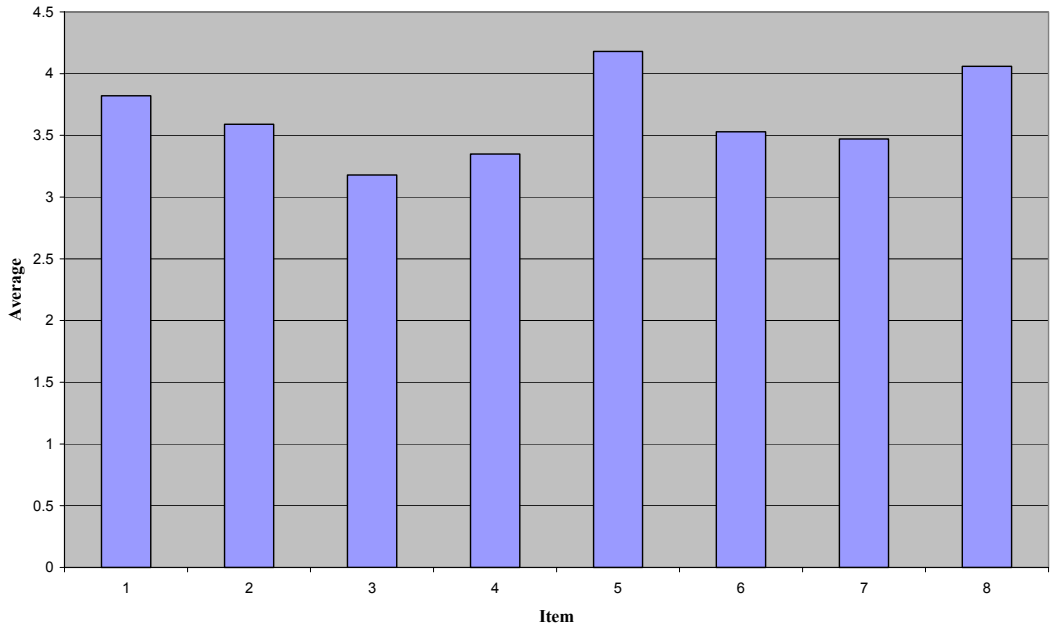
Phase IV: Expanding the results to the teachers in the jurisdiction of supervisory area

The Jurisdiction of supervisory area 5 had the responsibility to operate on expanding the teacher network in the area.

Phase V: Evaluating

It is the last stage of the project of developing the lesson plans using the Lesson Study Approach. The seminar was held on the process of this stage for the summary of the operation. The implementations were presented in the form of exhibition and seminar on the stage, which was held on February 26, 2005 at the Faculty of Education, Khon Kaen University.

Table I**:** Showing the opinions towards the success of the workshop (April 19-23, 2004).



Note: the average scores are calculated from 1-5 Likert scale style (showing 1: strongly disagree,
2: disagree, 3: indifference, 4: agree, 5: strongly agree

Items

1. The clarity in objectives of the project
2. The co-operation of the Faculty of Education, Khon Kaen University, Plan Organization and the jurisdiction of supervisory of Khon Kaen.
3. The confidence in putting theory into practice
4. The confidence in giving knowledge to others
5. The belief in developing the quality of teachers with the improvement of lesson plans
6. The belief in developing the quality of teachers with this approach being enhanced
7. The belief in developing the quality of teachers with this approach bringing long lasting results.
8. The belief in developing the quality of teachers with this approach influencing the quality of children in the future.

CONCLUSION AND REFLECTION

Problems and limitation

1. Problems of implementation

1.1 The teacher lacked of confidence specifically in linking the students' concepts to the teacher's expectation. This made the teacher feel frustrated with their role of teaching.

1.2 There are times when the teacher interfered too much with the students' thinking, causing the teacher's leading circumstances. The students therefore had no freedom to think as they should.

1.3 The students had no chance to discuss with other groups. They had no opportunity to learn to solve other students' problems.

1.4 The Open Approach is a time-consuming innovation. The Mathematics class usually takes only one hour. This caused the discontinuity of the activities resulting in the students' problem-solving.

1.5 The teacher prepared all he materials for the students but the students were not responsible for them.

1.6 It was difficult to group the students. There were forty-three students and the classroom was very small.

1.7 There was the difference in the group of student. Some cannot read.

1.8 There was a controversy among the teachers on "the accordance between lesson plans using Open Approach and the national test". Some teachers thought they were in accordance, other did not think so. The implementation of the plans, therefore, was not successful as it should be.

1.9 The network teachers, who came to observe the Open Approach teaching-learning, did not understand the model and the methodology of the approach. Unnecessarily, they interfered with the students' thinking process during the activities.

1.10 The teachers who used the Open Approach in their teaching were eyed and criticized by the authorities and colleagues in the school. This made them feels unconfident, pressured and uncomfortable to keep on using the approach.

2. Problems and restrictions of the follow-up

2.1 In case that there was a change of appointment time due to some urgent and unexpected work, contacting with the teachers was difficult. Some teachers gave the telephone number that cannot be contacted.

2.2 The teachers did not receive the returned plans sent by post from the committee, due to the ineffectiveness of the administration staff in the Faculty of Education or the post office. Additionally, the difficulty in communicating by phone made it inconvenient for the committee to give advice.

2.3 The cancellation of the follow-up appointment due to urgent work or other activities of the school or education regional area, such as the external evaluation of the office of national standards and quality assurance, mathematics Camp, or special events etc. This directly affected the timetable of the committee.

2.4 There was a misunderstanding of the coordinator, who was in charge of the school maps and the permission form to work outside for the supervisors, which delayed the follow-up.

2.5 The location of the school on the map did not match with the actual one. The journey was in the wrong direction and was delayed.

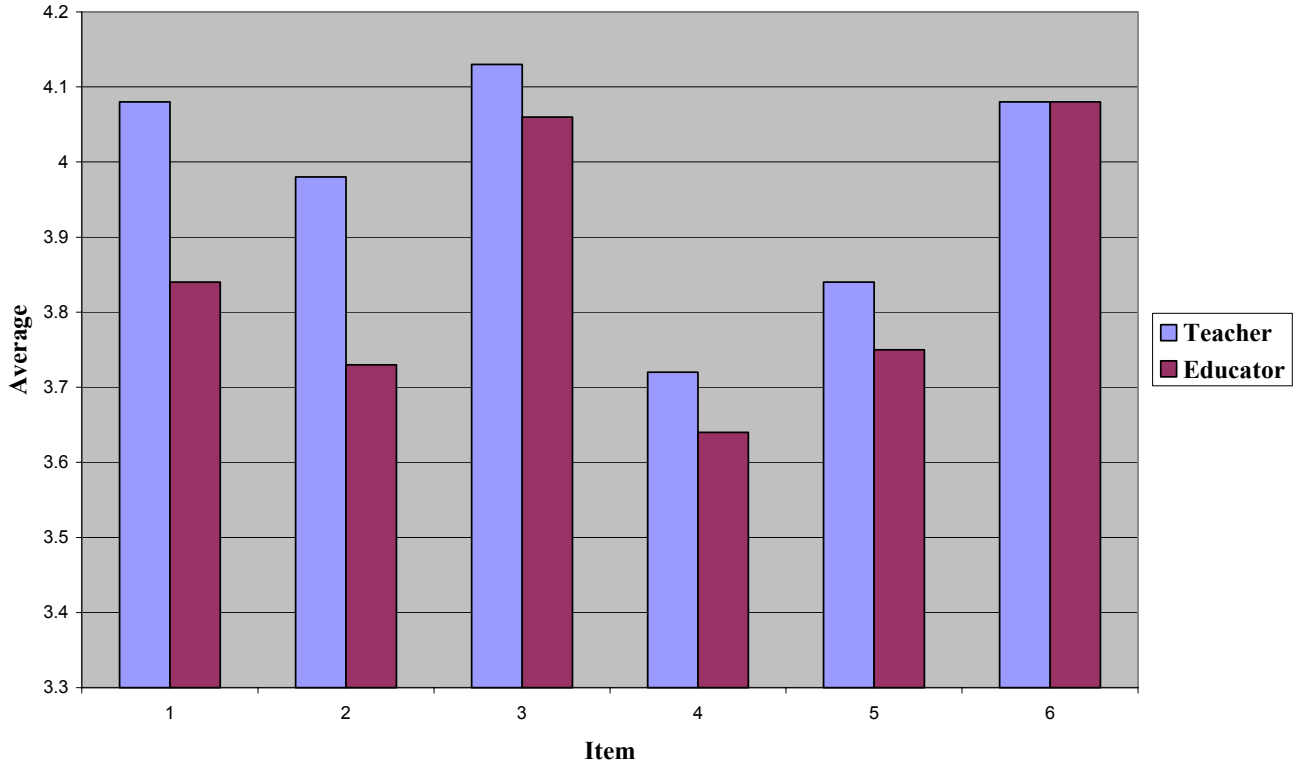
2.6 The supervisors were not free on the same day as they have lots of work to do. The transportation was hired too many times. Each follow-up could cover only a few classes.

2.7 Some schools were located quite far away from others, in bad condition roads, making the committee unable to go to all schools in the project. The follow-up to the remote schools was costly.

Reflections from the teachers

1. The teachers did not have enough time to prepare and write the lesson plans using Open Approach due to too much work other than teaching.
2. Writing this kind of lesson plans needs brainstorming to acquire diverse aspects. There must always be reflective thinking to achieve good lesson plans. Practically, it was almost out of question to get the teachers together because of such restrictions as the long distance among the schools, the high expense of traveling, the free time of the teachers and the teachers' other missions.
3. The teachers still did not understand their role in using Open Approach in their teaching. They did not know what exactly they had to do so they lacked confidence in teaching. For example, how to begin the lesson, how to sum up, how to write the evaluation form, what they were doing was right or wrong etc.
4. There was a doubt whether each unit of mathematics content could be written in Open Approach lesson plans or not, and how?
5. The staff from the regional education area did not at all take part in either the follow-up, or the project support after the workshop.
6. The results of the implementation of the plan shown that the students could think more, and sometimes the students could think more than the teachers.
7. The teacher had worried about being observed but after the observation was done, they felt that this is very supportive.
8. The teachers had more confidence in using the lesson plans in the class. They felt like having the supervisors with them.
9. This was the first project with the follow-up and gave the teacher assistance closely. The workshop was worth the time and it was the great activity.

Table III: Showing the average of the level of satisfaction with the objectives of the project.



Note: the average scores are calculated from 1-5 Likert scale style (showing 1: strongly disagree, 2: disagree, 3: indifference, 4: agree, 5: strongly agree

Item

1. The distribution of the teaching innovation
2. The understanding in the development of learning due to the education reform 1999
3. The teachers' capacity promoting due to the education reform 1999
4. The sharing of the learning development of the teachers living in the North-Eastern region
5. Forming the group for creating the education development in the community
6. Improving students' ability in studying by the learning innovation created.

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STRATEGIES FOR ADDITION AND SUBTRACTION OF WHOLE NUMBERS EXTENDED TO NUMBER SENTENCES INVOLVING FRACTIONS AND DECIMALS

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In this lesson study, Year 6 students (age 13 years) are asked to simplify, "using any appropriate strategy", eight addition and subtraction problems. The teacher also asks students "to communicate your thinking in your working".

WHAT DOES THIS LESSON SHOW?

1. When asked to simplify and solve sentences of the form $a + b$ and $a - b$, students have to think deeply about the meaning of "sum" (or total) in the case of $a + b$, and about "difference" in the case of $a - b$. (The expression "sum" is frequently used by teachers and in curriculum documents in describing sentences of the form $a + b$. The expression "difference" is less common. "Difference" is an important word because it expresses a relation between two numbers.)

2. This teacher uses the expression "*difference*" or "*find the difference between two numbers*" when asking students to consider sentences of the form $a - b$. Many other teachers would read a sentence of this form as "taking b away from a", or as "subtracting b from a", or even as "a minus b". While these are correct expressions, they express an operation, not a relation. The teacher's careful use of the relational word "*difference*" will be shown in the lesson.

3. The lesson is important because it builds upon ideas of relational thinking that the teacher has taught this year, and last year to those students who were in his Year 5 class. The nature of relational thinking, in contrast to computational or algorithmic-based thinking, will be shown in this lesson.

4. The lesson begins with number sentences involving only whole numbers, but in the second half of the lesson students have to think more deeply about addition and subtraction involving fractions and decimals.

5. Students apply strategies they have used to simplify addition and subtraction sentences with whole numbers to sentences involving fractions and decimals. Here, students have to think deeply about the nature of decimal and fraction numbers.

6. In discussion surrounding the eight questions, students are asked to communicate their reasons for choosing a particular strategy, and are asked to consider if there may be alternative strategies. Students are asked to comment upon strategies used by other students even though they may not have thought of using these strategies themselves.

7. Most of the strategies discussed by the teacher with the students are intended to train students to see that the numbers in sentences such as $a + b$ and $a - b$ are open to dimensions of variation (Marton & Morris, 2002; Marton, Runesson & Tsui, 2004),

and that recognising and using these degrees of variation is an important tool in simplifying and solving number sentences.

8. Helping students to see dimensions of variation in addition and subtraction sentences involving whole numbers, decimals and fractions builds up algebraic thinking. Seeing that numbers within number sentences can vary is intended to provide a bridge between the study of arithmetic and the emerging study of algebra.

9. The teacher's focus is having students see possibilities for simplification and successful calculation, and being able to explain why the simplification is appropriate. The teacher recognizes that some students will prefer to use strategies (i.e. algorithms that these students have learned in Years 2, 3 and 4) that do not rely upon relational thinking. All students are encouraged to consider other possibilities.

COMMENTARY ON THE LESSON RELATING TO EACH PROBLEM

Prior to the lesson, students were given an assignment entitled: Addition and subtraction strategies. They were asked to "Simplify using any appropriate strategy. Communicate your thinking in your working" for the following problems.

Problem 1: 826 – 489

The teacher commences by asking students, "What should you be thinking about before you start trying to work it out?" Some students say that they need to simplify the problem. The teacher asks again, "What are we trying to find?" Several students refer to a "difference". The teacher emphasises this point: "Any time we have a subtraction problem we are finding the *difference* between the numbers", then asks: "How have you thought it out?" A student says that he added 11 on to both numbers:

826 – 489
+11 +11

Before proceeding with the calculation, the teacher again asks, "Why did you add 11 to both numbers?" A student replies, "If you add eleven to the first number it makes the first number eleven more, so you have to add eleven to the second number". The teacher helps to finish the sentence by saying, "In order to …". Another student completes the sentence, "It keeps the *difference* the same".

The teacher asks, "Is there an alternative to adding eleven to each number?" A student says that it is possible to subtract 26 from both numbers, but agrees with the teacher that this would not result in making the problem easier.

The problem is completed, using an equal sign under the above working:

= 837 – 500
= 337

The teacher concludes by saying, "I know there are a number of you that are not using this strategy yet, but you can see [what it means]."

Problem 2: 9124 + 6968

Again the teacher asks students to think about what this problem is asking them to do. One student replies that it is asking them to find 6968 more than 9124. The teacher says, "We are finding the sum".

Students described how they simplified the problem: "Add 32 to the second number". The teacher explains, "Adding 32 to the second number makes the *sum* 32 more than it should be, so we have to take 32 from the 9124, giving 9092".

9214 + 6968
– 32 +32
= 9092 + 7000
= 16092

One student explains that it is possible to add 1000 first to the 9192 and then add 6000, giving the same result. The teacher accepts this possibility. He then concludes, "The reason why we are using these strategies is that they make it easier".

Problem 3: 3004 – 1746

The teacher asks first, "Can we use the same strategy here as we did in the first case?" Students agree that the goal is to make the second number into "a nice round number". This gives rise to the suggestion that they could add 54 to both numbers (resulting in the second number becoming 1800). The teacher then asks, "Why do we need to add 54 (to the first number) and not take away?" Students agree that the *difference* has to be kept the same, giving:

3004 – 1746
+54 +54
= 3058 – 1800
= 1258

One of the students suggests that 3058 – 1800 could be simplified further by adding 200 to both numbers:

3058 – 1800
+200 +200
= 3258 – 2000
= 1258

The teacher accepts this suggestion, and goes on to ask, "Is there another way we can approach this problem? … Generally, we look at the *second* number to make it easier." He is hinting that students think about transforming the first number. A student suggests taking 5 from each number, giving

3004 – 1746
– 5 – 5
= 2999 – 1741
= 1258

The teacher asks them to think why *nine* is the easiest number to subtract from. He says that some students might set out 2999 – 1741 formally (in vertical form). Students and teacher go through the formal steps: "1 unit from 9 units; 4 tens from 9 tens; 7 hundreds from 9 hundreds; and 1 thousand from 2 thousand".

Problem 4: 4024 + 7659

The teacher commences: "We are back to addition. We are finding a *sum* or *total*. What might we do here?" Students suggest subtracting 24 from the first number:

4024 + 7659
– 24 +24
= 4000 + 7683
= 11683

Asking students to compare this strategy (for addition problems) to those used with subtraction or difference problems, the teacher asks, "With addition, do we need to focus on any one of these numbers? No. We have got the choice. You can focus on either one of them. As long as you make one a nice round number to work with". (This contrasts with the subtraction problems where it was agreed "generally to make the second number a nice round number".)

The teacher reminds students that the equals sign can be used only at the beginning of equivalent lines. This last comment is important for understanding a student who said that he added 1 to 7659 and subtracted 1 from 4024, and then added 4023 + 7660 "bit by bit". The teacher asked, "You didn't use the equals sign (to connect lines) did you?" The student said he didn't. He said that he worked out the thousands first and then calculated the other parts of the sum "bit by bit":

4000 + 7000 = 11000
11000 + 600 = 11600
11600 + 60 + 23 = 11683 (Note that equals signs is used only to show results.)

Problem 5: $\mathbf{7\frac{1}{6} - 3\frac{5}{6}}$

The teacher and students note that they are finding the difference between the numbers. The teacher asks, "Which number are we going to focus on - to make it a nice round number? Not that you can't use the first number. But it's generally easier the other way". This leads to agreement to add one-sixth to both numbers, giving

$7\frac{2}{6} - 4$
$= 3\frac{1}{3}$

One student says that he did it another way. He said that he converted seven and one sixth to six and seven sixths. The teacher asks him to explain. He says that he converted one of the "wholes" in 7 into six sixths, giving

$6\frac{7}{6} - 3\frac{5}{6}$

$= 3\frac{2}{6}$

This student understands that he is taking away 3 wholes and also taking away 5 sixths. After hearing this second approach, the teacher adds, "You have so many alternatives. You don't need to do it one particular way."

Problem 6: $12 - 7\frac{4}{9}$

Here the teacher does not remind students that they are dealing with a difference. He asks what number could be added to both numbers in order to simplify the problem. Students see that adding five ninths is the best way to simplify the problem:

$12 - 7\frac{4}{9}$

$= 12\frac{5}{9} - 8$

$= 4\frac{5}{9}$

The teacher then says: "Some of you could work this (problem) out in your heads. But what goes down on paper communicates how you thought it out". Taking a lead from this comment, a student explains his approach as follows:

$12 - 7 - \frac{4}{9}$

$= 5 - \frac{4}{9}$

$= 4\frac{5}{9}$

Teacher adds, "Fine. You have realised that you have to subtract the seven *and* the four ninths".

Problem 7: 8.23 – 3.67

The teacher first asks students to explain the meaning of each of the decimal numbers. "What is .23?" Students: twenty three hundredths. "What is the 2 on its own?" Students: two tenths. "What is the 3?" Students: three hundredths. Then the teacher continues: "We are finding the difference again. What is the easiest number to focus on?" He adds quietly, "That is an interesting question", knowing that some students will take this as a hint that they might possibly focus on the first number.

Students agree that making the second number into a "nice round number" would be a useful simplification to start with:

8.23 – 3.67
+.33 +.33

= 8.56 – 4.00
= 4.56

Taking the teacher's earlier hint, one student suggests taking .23 from both numbers:

8.23 – 3.67
–.23 – .23
8.00 – 3.44

The teacher asks, "Is this going to be easier?" No one agrees. The teacher asks, "Could we make it even easier?", hinting that the above simplification has not gone far enough. Several students suggest subtracting .24 from each number:

8.23 – 3.67
–.24 – .24
7.99 – 3.43

"We are subtracting from *nines* again", says the teacher. Together the class repeats:

"3 hundredths from 9 hundredths, (giving) 6 hundredths."

"4 tenths from 9 tenths, (giving) 5 tenths."

"3 wholes from 7 wholes, (giving) 4 wholes."

Problem 8: 7.06 + 9.892

Reminding the class that this is addition, the teacher asks, "What number are you going to focus on?" Someone suggests adding .008 to 9.892 (to make it 9.900):

7.06 + 9.892
–.008 + .008

Without proceeding any further, the teacher asks, "Who will find this easier? Maybe, it's not such an easy strategy to use." The teacher adds, "If you start thinking to do it one way, you don't have to keep doing it that way". Some students see this as a hint to look at the first number. Some suggest subtracting .06 from the first number. The teacher asks them: "If you take .06 from this number, what do you have to do to the other number?" They reply that it is necessary to add .06 to the second number:

7.06 + 9.892
–.06 + .06

The teacher and students work together as they "add 6 hundredths to 9 hundredths", giving 15 hundredths. "From 15 hundredths, we can make 1 tenth and 5 hundredths".

7.06 + 9.892
–.06 + .06
= 7 + 9.952
= 16.952.

The teacher concludes the lesson: "[There are] many strategies to work. Textbook (formal) strategies are included. Sometimes it might be easier to use them. [You need

to ask] ‘What is going to be the best way of doing any particular problem?’ Now you have many strategies for doing that.”

HOW TYPICAL ARE THESE TEACHING APPROACHES IN AUSTRALIA?

In answering these questions, several points need to be made:

1. The school in which this lesson study has been captured is a private school and the strategies used by the teacher are not typical of many other teachers in the school. These teachers use computational approaches and, while encouraged to do so, have not introduced relational thinking into their mathematics classes.

2. The teacher in this study also gives attention to computational and algorithmic strategies for addition and subtraction. He is one of a growing group of elementary and junior secondary teachers who are moving arithmetic away from an almost exclusive focus on computational algorithms in order to foster students’ algebraic thinking. In this way, teachers are making a more effective transition between number patterns and relationships in arithmetic and some key ideas of algebraic thinking which students in the upper elementary school are expected to meet.

3. Australian national and state curriculum documents all recognize the importance of teaching students to use reliable written methods for computation. All curriculum documents also emphasize that mental computation has an important place along side the teaching of algorithms. There is no clear agreement among the various national and state documents about when formal written algorithms for the addition and subtraction of multi-digit numbers should be introduced into the elementary school mathematics program. Some states appear to favour a later introduction than others.

4. Authors such as McIntosh (in press) argue that mental computation should be given greater priority in the elementary school curriculum, stating that “when children calculate mentally, they use conceptual understanding of the numbers and operations involved, unlike the use of formal algorithms, which draws on memory of rules” (p. 4). McIntosh even argues “in favour of placing mental computation, instead of written computation, at the heart of primary school computational work” (p. 3). This position is more radical than that recommended in any of the national and state curriculum documents. McIntosh is quite careful to point out that his position is quite different from “advocating an even heavier diet of mental computation and tests which have formed the main approach to mental computation in the past” (p. 3).

5. McIntosh’s work has attracted the attention of national and state governments in Australia. In one national funded project led by McIntosh, the aim was to explore “in Grade 2, 3, and 4 classrooms ... ways of moving from mental computation to informal written computation, while refraining from teaching the formal written algorithms of addition and subtraction, and about the effects on teachers and children and their advice to teachers in other schools as a result of this project.

6. National and state curriculum documents no longer advocate priority of place to the teaching of formal written algorithms for addition and subtraction, although these approaches are still recommended in most of the national and state documents.

WHAT IS SPECIAL ABOUT THIS LESSON STUDY?

While national and state documents all recommend the use of mental computation and the teaching of alternative written methods, there is no one recommended approach. All recommended approaches, however, deal only with addition and subtraction of whole numbers. This teacher encourages students to use relational strategies to simplify and solve addition and subtraction problems involving whole numbers, decimals and fractions.

Other teachers who use a similar approach to addition and subtraction have found it especially helpful to some students who are experiencing difficulty in carrying out formal written algorithms. This is a very important point. While some students being taught in this lesson study may appear to be quite able, it should be remembered that the whole class is a mixed ability class, with some students who find mathematics difficult. Other teachers report that the strategies used in this lesson are accessible and successful with many students in Years 4 to 8 who are not mathematically able.

What does this approach have to offer to teachers and students in other countries, especially those APEC countries who are striving to build up the proportion of students entering and successfully completing secondary education?

These reforms require a rethinking of the elementary mathematics curriculum. In the past, because participation in secondary education was very limited, the elementary school mathematics curriculum focused almost exclusively on computation proficiency and the teaching of algorithms. This approach is unlikely to provide the growing proportion of young people entering secondary school with the mathematical experiences needed to understand algebraic structure and reasoning in the secondary curriculum. If more young people are to succeed in secondary school, then they must leave elementary school with a deeper experience of relational (algebraic) thinking which can be developed through number sentences and operations.

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BEGINNING THE STUDY OF THE ADDITIVE FIELD

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An innovation project is developing in Chile, the Strategy of Consultantship to the School for the Curricular Implementation in Mathematics, that considers the mathematical activity as the study of problems fields. In this article a first grade class is described and analysed, carried out by a "consulting teacher" from this Strategy who is responsible for the training of other teachers. The training is carried out through the study and application of Didactic Units that are proposals to organize brief learning processes in those the sequence of the proposed tasks generates the evolution of the techniques and knowledge that the students put at stake.

THE STRATEGY OF CONSULTANTSHIP TO THE SCHOOL

The level of Basic Education includes eight years in Chile, and it has a covering about 97%. About 92.8% of the children study in schools subsidized by the State, administered by a Town council (52.3% of the registration) or by a private supporter (40.5% of the registration). The Ministry of Education regulates the curriculum, evaluates the students' performance and carries out diverse initiatives to improve the educational system, such as training and evaluation for teachers, distribution of study texts and other resources for the learning, and implementation of special programs for schools with poor academic performance.

When pupils finish their elementary school 4th grade (ten year-old children) a performance measurement for all students is carried out. Evaluating the result of these measurements, and considering the new existing Study Programs from 2002, the Ministry of Education began a Reading-Writing-Mathematics Campaign (LEM), dedicated to improve pupils'elementary learning in the first school cycle. According to this Campaign, an agreement was settled down in 2003 between the Ministry and the Chilean University of Santiago to develop a "Strategy of Consultantship to the School for the Curricular Implementation in Mathematics". During the first year, a pilot plan was carried out in twenty schools and, during the two following years, this strategy was applied in almost two hundred schools, in three Regions of the country.

The Strategy of Consultantship to the School elaborated a didactic proposal being based on the Anthropological Theory of Didactics (Chevallard, 1999) that takes mathematical activity as an activity of fields' study of mathematical problems:

> The mathematician does not only aspire to think about good problems and solve them, but rather he also seeks to characterize, to define and even to classify the problems in "types of tasks" to understand, describe and characterize the techniques that he uses to solve them, until the point of controlling them and regulating their use, he intends to settle down the conditions under which these are working or they are not applicable

> and, ultimately, he aspires to build solid and effective arguments that sustain the validity of his ways of proceeding (Bosch, M., L. Espinoza y J. Gascon, 2003).

According to this theory, so that children learn mathematics it is necessary that they deal with problems, elaborate procedures to solve it, explain and justify the operation of their procedures, students exchange and compare procedures between themselves, they are willing to adopt those that are more effective to solve the outlined problem, explain the knowledge that support them and relate them with knowledge that already have, then they will deal a new problem located in the same field that presents new challenges for the students.

According to the Theory of the Didactic Situations (Brousseau, 1990), in this proposal is considered that the "sense" of a mathematical knowledge is built facing with a set of problematic situations where this particular mathematical knowledge appears like a tool for their solution. These situations should allow children to elaborate strategies from previous mistakes, from the inadequacy or "fail" from their previous knowledge and the modification of the same ones.

The Strategy of Consultantship to the School elaborated a model to transform the pedagogic practices, based on sixteen Didactic Units, for the first four elementary grades. Under a consulting teacher leading, the teachers of each school study these Didactic Units and afterwards they apply them, with the observation and feedback of the consulting teacher.

The class that I will present was carried out in 2005, in a council school of a town located 500 km to the south of the country's capital. The students are considered vulnerable or with social risk factors, due to its economic lacks and incidence of activities of socially disintegrating character. The teacher who imparts the class is a "consulting teacher" since 2004. She is recognized as a good teacher by the local ministerial authorities; she has been worthy of a special salary assignment for her "pedagogic excellence" and she has been selected to evaluate other teachers through a mechanism of "elaboration of briefcases". She is also recognized by the community, since parents of their current students accepted they were changed from afternoon to morning period, so that she was the teacher of their children.

The elected class corresponds to the first class of the Second Didactic Unit for the first elementary grade, elaborated by the Consultantship Strategy team. The Unit is titled: "Additive Problems of Composition" and it is a proposal to organize the study of this topic during three classes, from 90 minutes each one. The Unit contains an outline that allows to visualize the learnings that it is awaited students achieve, the learnings that should have previously acquired, the progression of proposed tasks and the awaited procedures during these three classes, besides plans for the classes that describe the activities to carry out, working sheets for students and an evaluation instrument[1].

[1] See in Appendix: Outline of the Didactic Unit, plan for the first class and work sheets for the students first class.

Description of a good class

The sequence of activities of the class gets close to the plan elaborated by the Strategy of Consultantship to the School, as introduction to the study of the additive problems. For a self-controlling mechanism, the teacher has written the activities in cards that she reads aloud simulating they are Pepito's letters, a pelican cut off in bristol board and superimposed at the corner of the blackboard. Following the plan, the class consists of three well differentiated moments, an *initial moment*, where children exercise abilities considered as previous to the addition and subtraction learning, a *development moment*, in which they carry out and observe actions for joining and separating collections of objects, associating them to the addition and subtraction operations, and a *closing moment*, in where what was learned is institutionalized.

In the *initial moment*, the plan proposes:

- Say the numeric sequence in upward and descending form, at least up to 10. The teacher structures this activity forming a row of seven children and requesting them that they bend over; each one rises when saying a term of the sequence in upward order, and they bend over when saying it in descending order again. According to children's wish, the teacher accepts they also say the sequence from 0 to 6 and from 6 to 0.
- Count at least up to 10 objects. A Pepito's letter notices: "Children have not been counted". The teacher designates three children so that they count the members of their row. Numbers are registered on the blackboard, each of them digits. At Pepito's request, absent children are counted. The teacher reads their names and children put a stick on their tables for each name. The total sticks are 12.
- Read the numbers up to 10, and copy them from the numbered ribbon. At the beginning of the class two children order the first 25 numbers, written in cards with a rope. Once reordered, they read in a chorus. After counting the absentees, they identify the 12 in the rope, as well as the previous number and the following one. The teacher writes two digit numbers on the blackboard and children read them mentioning which it is bigger. At the initial moment children don't write numbers.

At the *moment of the class development*, the *mathematical task* specified in the plan is: "Determine the quantity of objects resulting from joining or separating actions, proposed verbally by the teacher, manipulating objects that are accesible to children. Numeric range up to 10."

According to the plan, the teacher organizes an activity in which children manipulate objects. In order to make activity more attractive, she includes a song about a hen, known by children. When saying each number, children put a stick on their table. The stories are:

- The hen put 3 eggs during the first day and 5 eggs the second. How many it put in the two days?

- Considering the 8 eggs it put, 3 were broken. How many are now?

In order to determine their answer, children count the objects that have on their table, awaited *technique* for this first class, according to the plan.

The teacher organizes another activity changing the *conditions of carrying out the task.* This time, the objects that join and separate -red and blue notebooks- they are only manipulated by the teacher. The children follow the actions visually and they determine, counting at distance, how many are they:

- 2 red notebooks plus 3 blue notebooks
- 3 red notebooks plus 3 blue notebooks
- 6 notebooks minus 3 blue notebooks (the same ones that were joined)
- 4 red notebooks plus 3 blue notebooks
- 7 notebooks minus 3 blue notebooks (the same ones that were joined)

In these exercises, the subtraction appears as inverse operation from addition, in which teacher enhances the proposal of the plan.

The plan intends to study the case of additions and subtractions in those the second term is 1. The teacher comments that they already know it, talking to Pepito. She continues manipulating notebooks:

- 4 red notebooks plus 1 blue notebook
- 5 notebooks minus 1 blue (the same one that was joined)
- 6 red plus 1 blue
- 7 notebooks minus 1 blue (the same one that was joined)

As last activity of this moment, the plan proposes for children 4 work sheets, with collections of drawn objects that can be separated in two subcollections. Children should count the objects in one or both subcollections and all the objects, sometimes answering in oral form and other in written form. In the first one, second and fourth work sheet, the objects are children or animals. In the third work sheet the objects are triangles and rectangles. The teacher draws an example of each one on the blackboard and she asks the children to identify them. When she reads that they should count the small rectangular figures, she comments: how difficult it is!

At the *closing moment*, the plan of the class indicates that the teacher should systematize what they learned, associating the action of joining collections with addition and the action of separating collections with subtraction. In order to determine the sum or the subtraction, the counting is used, as procedure. When one of the collections has only an object, the result is the following number, or the previous one. The teacher asks what they learned today. A girl says: I learned to subtract and to add and to do mathematics with the numbers. Another: And to count, with the sticks. Another: order the numbers, too. The teacher asks them to sing the Pepito's song, as they always finish the class, changing the letter to say what they learned today.

Why do I consider that this is a good class?

First of all, the structure of the class is consistent, it is adjusted to a proposal designed by a project specialized in the implementation of the national curriculum. The teacher segments the class in the three moments indicated in the plan and she is guided by the appropriate sequence of activities.

In relation to emotional environment, the teacher has a very significant support for children: Pepito, the clipped pelican. When feigning to dialogue with this character, the teacher introduces in the class an imaginative and funny dimension, full of surprises. It is necessary to order the numbers that Pepito disordered; it is necessary to sing "The Francolina hen" because Pepito likes it (and children too); it is necessary to sing the song dedicated to the pelican, to explain him what they learned in the class. This funny tone proposes the development of a positive attitude toward mathematics' learning.

The class is focalized in the mathematical task. The teacher introduces humorous comments and she accepts those from children, but she quickly recaptures the course of the work, giving clear signs of what it is the important in order to achieve the learning.

The work rhythm is intense; hard-working time. The course of the activities is continuous; practically there are not interruptions. Occasionally an adult enters and goes out discreetly, without distracting the group. The teacher has a voluntary assistant, the mother of a student, who distributes materials and assists some children with basic necessities. In certain moment, a girl comes closer to the teacher for asking sticks; the teacher is looking for the sticks and then gives her, while she continues giving instructions to the group.

Along with adopting a plan that has not been created by her, the teacher carries out a class underlined by her personal style. Using the didactic proposal of the Strategy, she uses a margin of professional freedom to implement it. In order to solve tensions between the proposed plan and her appreciation about the students' competence, the teacher attributes Pepito the responsibility of the plan. The Pepito's messages constitute a means of controlling the program's execution but, at the same time, it is possible to qualify some activities like too easy: "¡but if everybody already knows it, Pepito! ", or very complex: "how difficult is this!". Although she practically carries out all the proposed activities, introduces variations: she enlarges the numeric range of 10 to 25 for ordering and reading of the numbers, she only manipulates the objects in some of the actions that generate additive situations, and she separates the same collection of objects that previously had joined, proposing this way an intuitive anticipation of the inverse character of the addition and subtraction operations.

The teacher seems receptive from children propositions. She accepted their restlessness for the zero, allowing them to say the numeric sequence from this number, besides saying it from the 1. Also, she invites them to write additions on the blackboard, since some already know it, although the plan proposes they only write the result. She corrects them, introducing the equal sign, and she shows them how to write a

subtraction. She is interested to know where they learned what they know and she stimulates them to share their knowledge during the class, without moving away from her own plan.

The children keep up expectant. They are happy, relaxed, and willing to carry out the proposed tasks. Teacher encourage children to express themselves openly, as much responding to her questions as expressing their emotional reactions and communicating their appreciations concerning any topic. This opening is propitiated by the supposed spontaneity of Pepito's behavior. Indeed, through Pepito, the teacher verbalizes infantile necessities and emotions, during the class. Pepito carries out mischiefs, feels happy or sad, shows with vehemence its impulses and the teacher feigns to dialogue with him to become calm, instead of doing directly with her students. In order to avoid conflicts, the teacher attributes complicated decision making to Pepito, for example, what boy goes to the blackboard in a given moment.

The relationship between the teacher and the children contains as much guiding elements as permissiveness. The teacher, encouraging them with Pepito's tales, manages the class; this management is accepted by all the children. On the other hand, she is able to establish a grade of trust that allows children to express themselves confidently, saying what they know and what they don't know, or something that other boy told them, generating a climate of frankness and intellectual honesty.

Finally, there is a 40% of absent children. Although the teacher recognizes that this situation "make Pepito unhappy", she does not get discouraged and she works with the current children with the same enthusiasm that, without a doubt, she would deploy if she had complete attendance.

What kind of reform is expected, with practices as the one observed?

In the observed class it is operating the didactic proposal elaborated by the Strategy of Consultantship to the School. The class is part of a Didactic Unit where it is proposed a planning for several classes, at the end of which it is awaited that the students achieve certain learnings. Through these classes, the students face different types of mathematical tasks, they should elaborate procedures or techniques to undertake these tasks. The tasks, they are proposed by the teacher, but the techniques arise from what children know or can discern, in the moment to undertake them. Once the task was carried out, the teacher manages a discussion in which children expose their techniques and they compare them, according to the effectiveness regarding the proposed task, in order to choose one of them for the use of the whole community.

The tasks and their execution techniques constitute the practical component of the mathematical activity, but the school study would not be complete if it have not been included its theoretical component. Once children have a technique to carry out certain task, this late is modified changing their conditions to carry out, so that children will have to change the techniques which they undertake. The simple task of "add 1", can be replaced with "add to 1", requesting children determine the quantity of notebooks when the teacher puts 1 red notebook and then 6 blues. The technique of saying the

following number of the first term is no longer useful and it is very probable that the children need to count all the notebooks to give the answer. The commutativity resource will arise but, how to justify this property?

Along the Didactic Unit, starting from the sequence of proposed tasks and from the evolution of the techniques used to solve them, the teacher opens the discussion about the progress in mathematical knowledge of the group. The discussion about techniques, their reach (where they work) and their justification (why they work), it corresponds to a more theoretical level of the mathematical work that is carried out in the classrooms. The teacher stimulates the search of relationships among the used techniques and he guides the formulation of properties, concepts and theorems. Walking towards the theory, it supports the search of relationships among the acquired knowledge.

The reform of the teaching and learning practices that is expected, begins with the study and application of these Didactic Units by the teachers of a school, under the management of an experienced teacher and previously qualified in the frame of the Strategy, the "consulting teacher". The study begins with a process of problematization of teachers'knowledge in regard to the topic boarded in the Unit. The teachers face a problem or mathematical task and they solve it with the techniques they manage, later identifying the mathematical knowledge they put at stake or those they have reformulated, or acquired, during the process. Then they read the Didactic Unit that contains, besides specific proposals of activities to organize the work with the children and evaluate the achieved learnings, an extensive chapter to argument, from a didactic and mathematical perspective, the curricular decisions made by the authors of the Unit.

The following step for the appropiation of the didactic proposal, consists of applying the Unit studied in the course where each teacher teach. In this process, teachers are accompanied by the consulting teacher who attends some of their classes as participant observer, subordinating his participation to the support needs expressed by the teacher that manages the class. Afterwards, the consulting teacher gives feedback to the teacher who applied the Unit, in individual and in collective sessions, with the participation of all the teachers of the school. In these sessions it is very useful to have videorecords of the classes, in order to have a repeated observation and an analysis more objective.

Besides training teachers starting from the study and application of four Didactic Units in a school year, the consulting teacher interacts with the directive personnel of the school, in order to generate institutional conditions that support the study and appropiation of the didactic proposal by the teachers.

The Didactic Units contain propositions to organize the educational work during a brief period, one or two weeks. They foreshadow a learning process, providing a basic structure that needs to be complemented by each teacher. It is in this complementation where the teacher's master is evidenced since his class has been selected as example of a good practice. Besides modifying some of the proposed activities, she puts at stake her knowledge about how to delight and to make their students work. When operating

through consulting teachers, the Strategy of Consultantship to the School aspires to enrich its proposals with the exchange of experiences among the group of teachers that are beginning the study of the Didactic Units and an experienced teacher that has already studied them and perform some of them with their students, as in the case of the consulting teacher whose class we have analysed.

The Strategy of Consultantship to the School proposes fundamental changes in the paradigm that operates in our national educational system at the moment. It intends to change:

- A teaching focalized in the learning of concepts and mathematical procedures, to be changed by a teaching based on the study of problems.
- A teaching of isolated concepts, to be changed by the undertaking of articulated problems fields.
- The presentation of definitions and explanations from the teacher or from a text, to be changed by the collective construction of senses and mathematical meanings, assumed as a cooperative task.
- An activity few established, to be changed by other, based on arguments and justifications arised from the own children work, and that respects the consistency and mathematical rigourness.

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APPENDIX

1. Outline of the Didactic Unit
2. Plan for the first class
3. Work sheet 1
4. Work sheet 2
5. Work sheet 3
6. Work sheet 4.

SECOND DIDACTIC UNIT:
Additive Problems of Composition

AWAITED LEARNINGS FROM PROGRAM

- *They associate the addition and subtraction operations with the actions of joining or separating sets and adding or removing objects, in situations that allow determining unknown information from available information. (Awaited learning 5, first semester).*
- *They manage mental calculation of additions and simple subtractions in the range 0 to30. (Awaited learning 6, first semester).*

AWAITED LEARNINGS FOR THE UNIT

- They associate the addition with joining objects from two collections in only one.
- They associate the subtraction with separating objects from a collection in two collections.
- They solve additive problems associated to the actions of joining objects from two collections in only one or separating the objects from one collection in two.
- In some cases of additions and simple subtractions, they add or subtract for evoking results, that is, through mental calculation.

Previous learnings:

- Say the numeric sequence in upward and descending form, at least up to 10.
- Count objects, at least up to 10.
- Identify each one of the numbers up to 10 and copy them, for example, from a numbered ribbon.
- Locate a well-known number from the numbered ribbon, and continue saying the numeric sequence to locate the writing of another number.

Central ideas for this unit:

- Counting is a procedure that allows determining the quantity of objects that result from joining objects of two collections or separating objects of one collection in two parts.
- The action of joining objects from two collections in only one, it is associated to addition.
- The action of separating objects from one collection in two collections, it is associated to subtraction.
- Addition allows anticipating the quantity of objects that will result from joining objects of two collections.
- Subtraction allows anticipating the result of separating objects from one collection in two parts.
- In some specific cases, it is possible to anticipate the result of the actions through mental calculation

Traverse objective:

It will tend to develop in boys and girls, the self-confidence in the own possibilities of solving problems that imply results about certain actions.

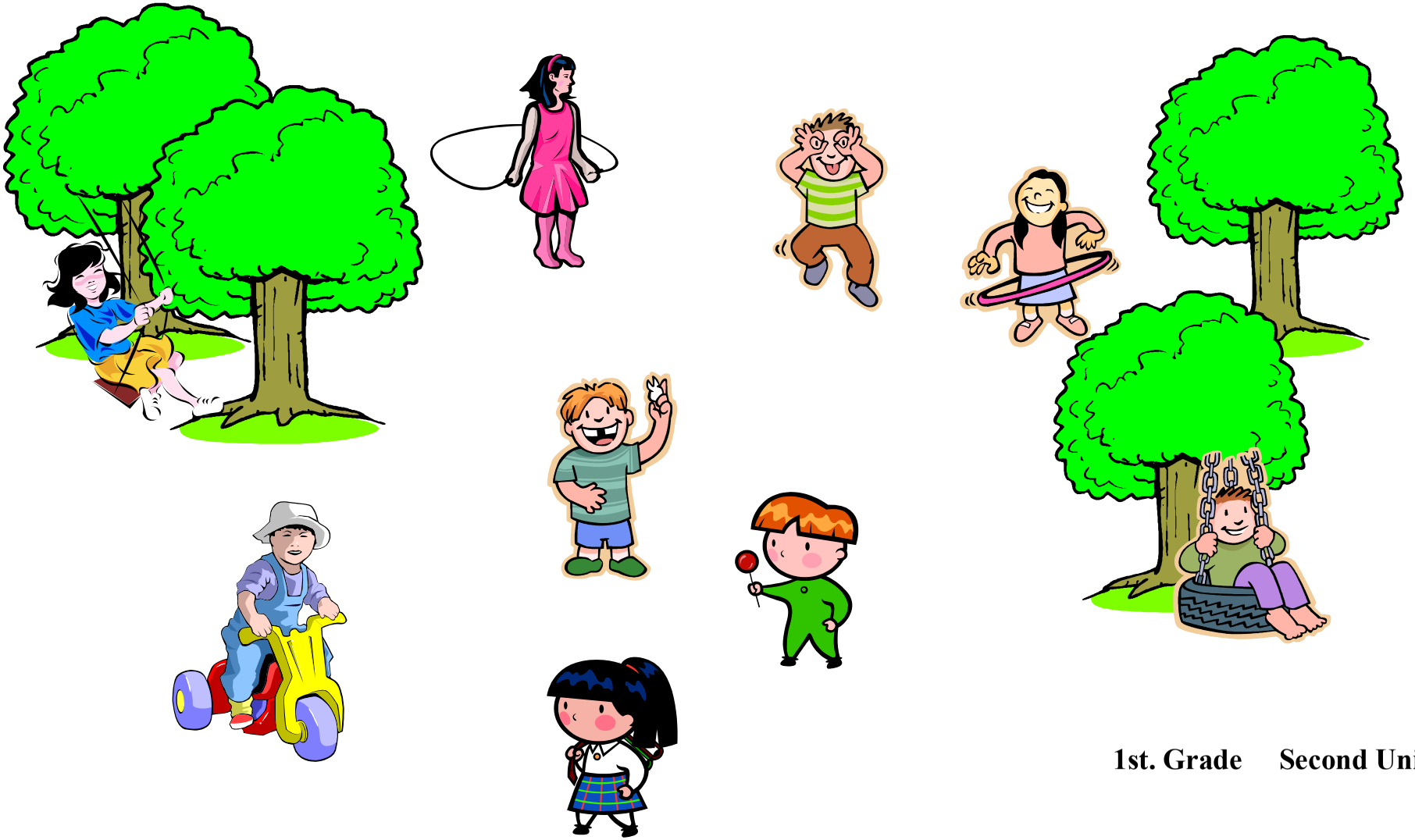
Plan for the first class (90') Solving composition problems, in the range 0 to 10
Materials: Objects in the classroom: school tools, colors cards, sticks, bottle covers, pupils, etc.

Mathematics Task	Activities	Evaluation
They determine the quantity of objects that results from joining or separating actions, that the teacher verbally proposes, manipulating objects that are close to them. Numeric range up to 10.	**Starting the class. Collective situation:** The teacher exposes problems to children that allow him to be sure that all of them know: - to say the numeric sequence in upward and descending form, at least up to 10 - to count at least up to 10 objects - to read the numbers up to 10 and to copy them from the numbered ribbon **For example:** - he asks a boy to begin the numeric sequence and others to continue this sequence up to 10; then he request other children to say the numeric sequence in descending form up to 1 or 0 - questions: how can we know how many children are there in this row?, how many are?, how many books are there on my desk?, etc.	The children performance according to appropriate tasks can suggest the teacher to propose activities like: say the sequence in descending form, count objects distributed in different forms, locate and copy numbers from the numbered ribbon.
	Development of the class. Collective situation: For each problem that the teacher exposes, children manipulate the objects according to the proposed problem: they join or separate objects, according to the case. They always should give a complete oral answer and write the corresponding number. In relation to the relationship among the numbers, they are of two types: 1. Some problems in which one of the collections has an object: **Example 1:** Ana has 7 sheets and Luis has one. They join their sheets. How many do they have? **Example 2:** Ana and Luis join 8 sheets. Ana has 1. How many does Luis have? 2. Others problems, they could include any couple of numbers (whose sum does not overcome 10): **Example 3:** There are 6 small books and 3 big books on the teacher's desk. How many books are there on the desk? **Example 4:** There are 9 books on the teacher's desk, 6 are small and the other ones are big. How many big books are there? The teacher asks in each case: what are we doing with the objects?, what operation are we doing with the numbers?, how are we going to find the result?	It is important that everybody join and separate objects correctly. There will be children that don't need to count all the objects; it is not necessary to be demanding with them. The different levels of students' performance will suggest the teacher to propose problems in a bigger range of numbers; joining or separating problems, etc. It is important that all children give a complete oral answer.
	Work Sheet Task: Students work in work sheets 1, 2, 3 and 4. The teacher can add activities to those proposed in the work sheets.	Some students will need concrete material.

Closing the class. At the end of this class, the teacher should systematize what they learned: to solve problems, a procedure is adding (if they have joined objects) or subtracting (if they have separated objects) through counting. In the cases in that a collection has only one object, the result is the following number or the previous number, according to the case.

Work Sheet 1 Name: _____

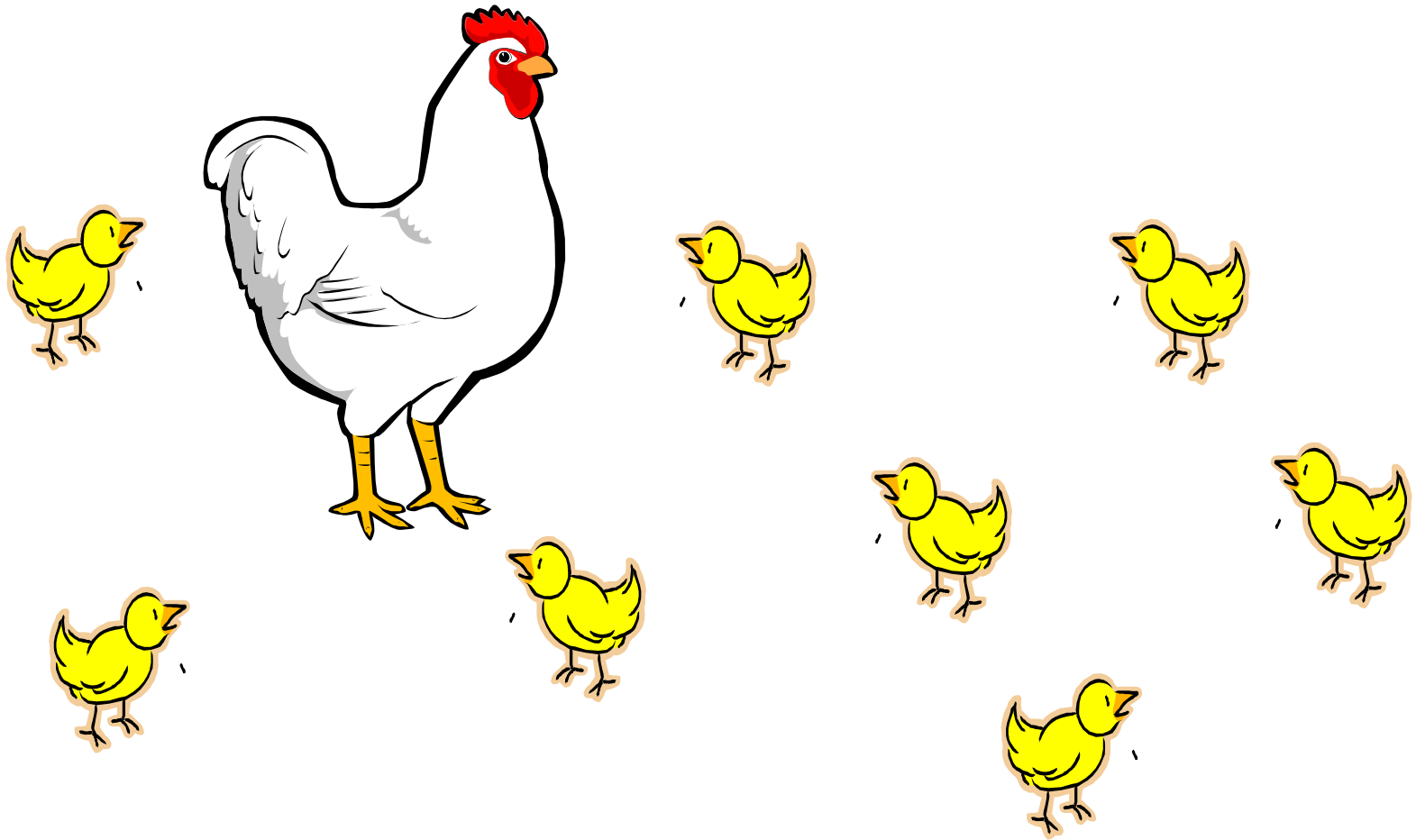
Answer verbally: How many girls are there in the square? How many boys are there?
Write into the box the total amount of boys and girls that there are in the square.
You can copy the number from the numbered ribbon.



Work Sheet 2

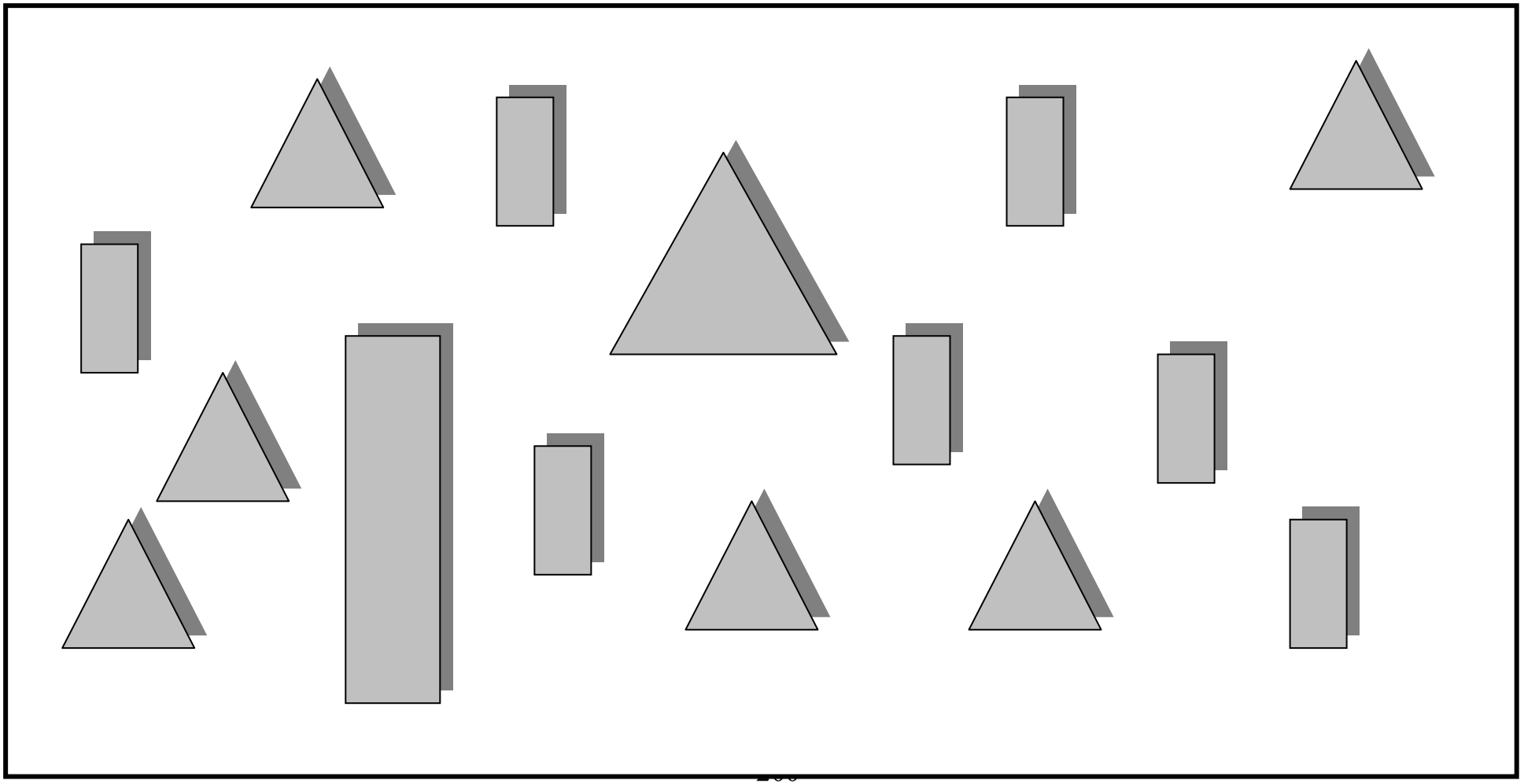
Name: _____

Respond verbally: How many birds are there on the sheet?
Write into the box how many chickens there are.
You can copy the number from the numbered ribbon.



Work Sheet 3 **Name:** _____

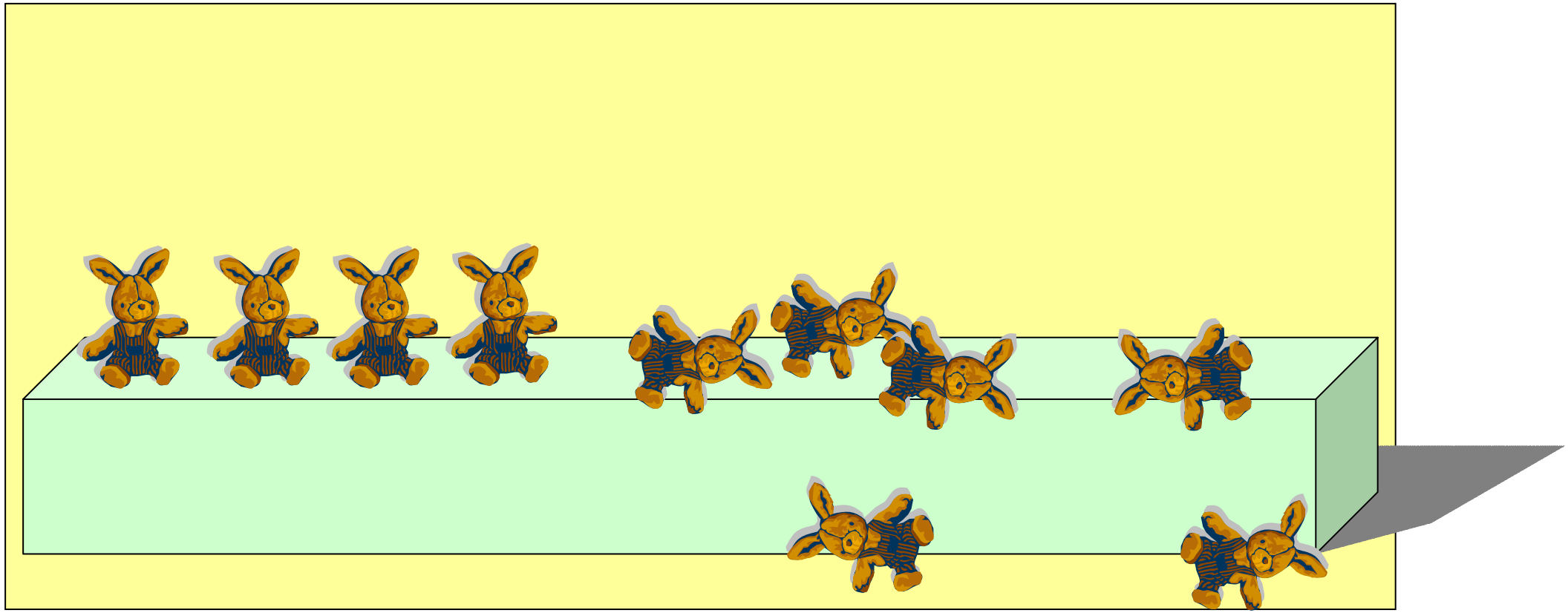
Inside the frame there are rectangular and triangular objects.
Answer verbally: How many rectangular small objects are there into the frame?
How many small triangular objects are there?
Write into the first box how many rectangular objects there are
Write into the second box how many triangular objects there are
You can copy the numbers from the numbered ribbon.



Work Sheet 4

Name: _____

In a game 10 bears were placed and Ana made the first play.
Answer verbally: How many bears are standing?
Write into the box how many bears Ana throw away in her play.
You can copy the number from the numbered ribbon.



ON THE ENHANCEMENT OF CREATIVE & INDEPENDENT AWARENESS OF PRIMARY SCHOOL PUPILS

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Introduction

The first Vietnamese class of the secondary school for gifted pupils in mathematics was established 40 years ago, in 1965. Nine years later Vietnam participated in the 16th International Mathematical Olympiad (IMO) in Germany, and got 1 Golden, 1 Silver, and 2 Bronze prizes. It was rather good debut for the newcomer country in the history of IMO. Over the past 30 years, Vietnam was very proud of the high results at IMO. However, it seems that many of laureates didn't have high achievement in their university study, and therefore, they would not become scientists in the future as expected. It turned out that during many years the pupils have been trained in the way of "fighting-cock", which could not promote the creative ability in the next education process.

Certainly, to become a real scientist, pupils and students are required to have a lot of factors from ability of thinking, really creative aptitude to conditions, working and studying means, etc. One necessary thing considered is that how to cultivate the ability of independence and creativity for pupils from the first grades of the schools.

Therefore, it is necessary to start a process of innovative teaching and enhancement for primary school teachers, that makes changes in the awareness of teaching staff to self improve teaching quality, thereby to improve independent and creative awareness of primary school pupils.

There are different points of view on Education Reform so far. Many people agreed on the necessity of permanent and consecutive reform in the content of textbooks, which is suitable with the development of sciences and technology as well as with the development of historical and social issues. These people assumed that after graduating, pupils and students must be equipped with enough necessary knowledge which is suitable with the development of sciences and society. However, some others said that after each reform, their children became "testing mice".

Therefore, the aim of the process said above is to establish a standard program for primary school teachers, and thereby, to contribute to the enhancement of education quality at primary level.

Nowadays, in Vietnamese primary schools, even in Hanoi and Ho Chi Minh City, pupils are trained by the method "professor dictates, students write, and do exactly what professor says". A class is evaluated as a good one if all pupils silently listen to

their teacher, raise their hands in the right way when they want to present their ideas to establish this lecturer, etc. This regulation is applied in all schools for all pupils from the first to the last, fifth grades.

It's known that primary level is the basic level in the educational field. However, all pupils are trained in such a way that we unintentionally created a generation of machinelike pupils.

Following the program, at the beginning of January all pupils must take the examination for the first semester within one week with 9 testing disciplines. Certainly, after (learning) each discipline an examination is necessary. However, the existing issue is how to establish a good program for evaluating pupils' knowledge. If the testing program for the first semester for the first and the second grades is the same with the program for the third, the forth and the fifth grades, then all pupils will feel afraid and that method is unnecessary, or in some case it becomes "unscientific".

It turned out that in some primary schools, the teaching method is not pedagogic at all, and it doesn't stimulate pupils' independence and creative thinking.

In order to prepare for the next examination, all pupils must revise all their lessons under the instruction of their teacher. For Literature of the forth and the fifth grades pupils are instructed to learn by heart all revised texts, because the question should be one of them. To prepare for the exams of Maths of the first grade pupils, after finishing two classes at school, pupils must do their homework until 10 or 11 pm with 8 to 10 Maths exercises and 2 to 3 pages of writing exercises. Some of them become crying in learning. The above issue is one example of the best primary schools in Hanoi and Ho Chi Minh City. That is the problem of primary schools in big cites. The question is what about the status of primary schools in the whole country, or at least in the remote areas?

Therefore, it is necessary to start a project on knowledge enhancement for primary school teachers that establishes a standard program. Knowledge and skills are necessary criteria to improve comprehensive education quality in schools nowadays. These criteria link closely together in order to create the real quality for primary level, especially to train one independent, creative and active youth generation.

It's difficult to get out of all existing teaching method as well as of all thinking which become a bad habit right now. But we must change them. It's impossible to let all pupils to continue an obligatory, inflexible and uncreative curriculum. Certainly, pupils must be provided enough knowledge to become labors who have suitable capability in regional areas and in the world. Therefore, capability and knowledge of teachers must be improved in order to teach and communicate to their pupils. So, It's necessary for all teacher especially the first form teachers to self improve their level, knowledge on the surrounding world, at the same time make reference to other teacher's method though which teachers can build the most scientific teaching method for themselves in order to promote the independence and creativity of pupils, and to create a good habit just in the first form pupils.

Teaching and test using softwares

We will present a software "I Study Mathematics" for the first grade of primary schools.

There are 3 parts:

1) Numbers within 10
2) Numbers within 100
3) Relax with logic thinking

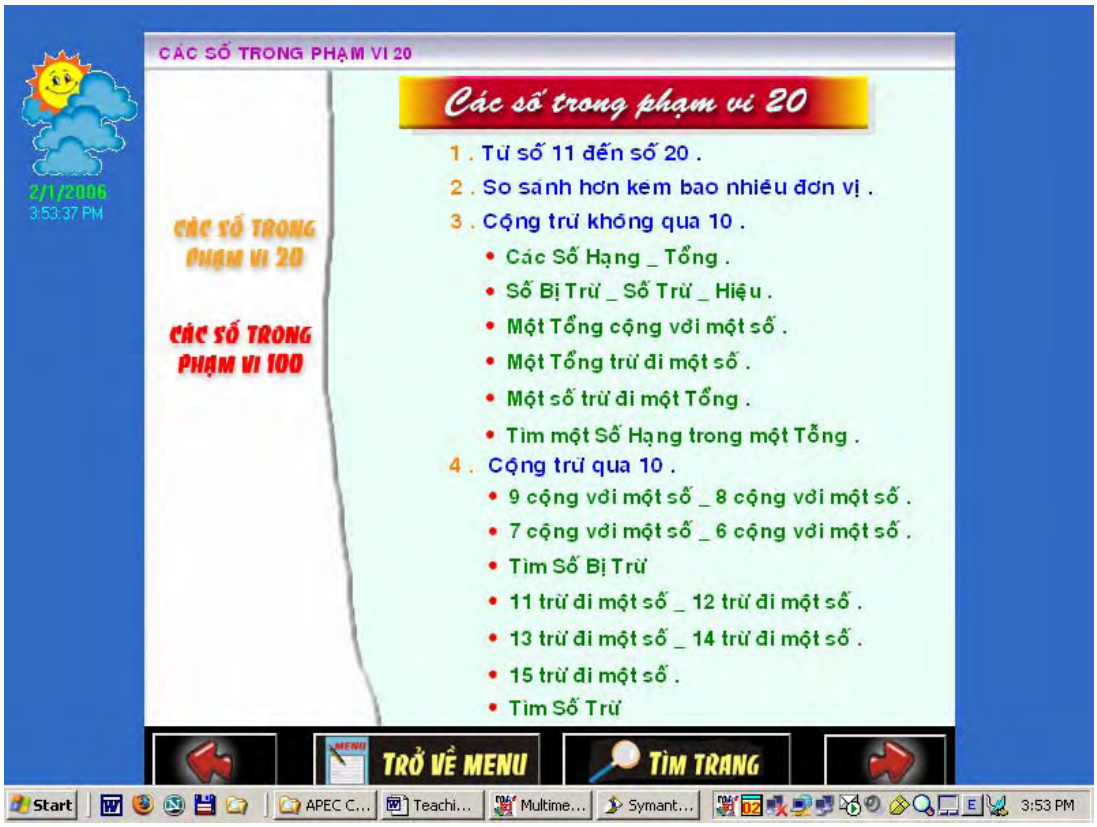
In the first two parts there are exercises/problems pupils should do/solve, and they can check their answers by clicking on the icon "View the result". Also pupils can redo these exercises/problems.

It is special with the third part. Here there provide different problems that require a logic thinking.

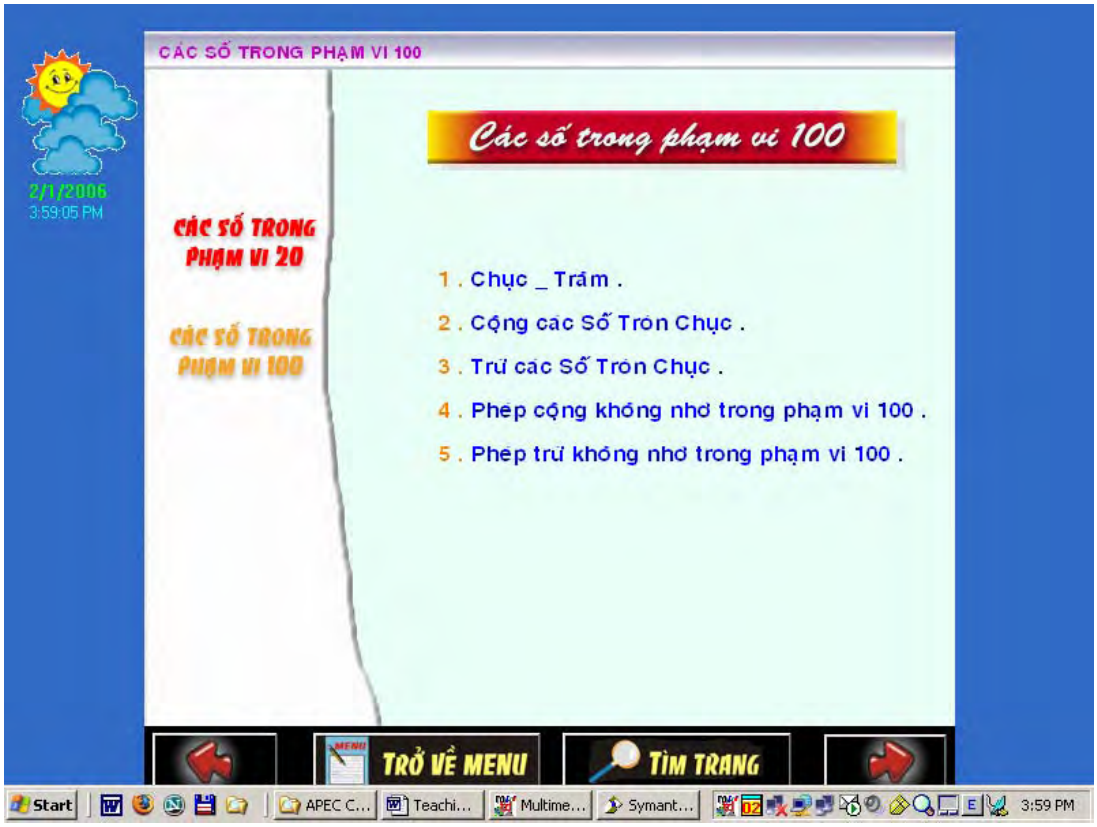
1. PART ONE: "Numbers within 10"

2. PART TWO: “Numbers within 20, and 100”

2.1. Numbers within 20

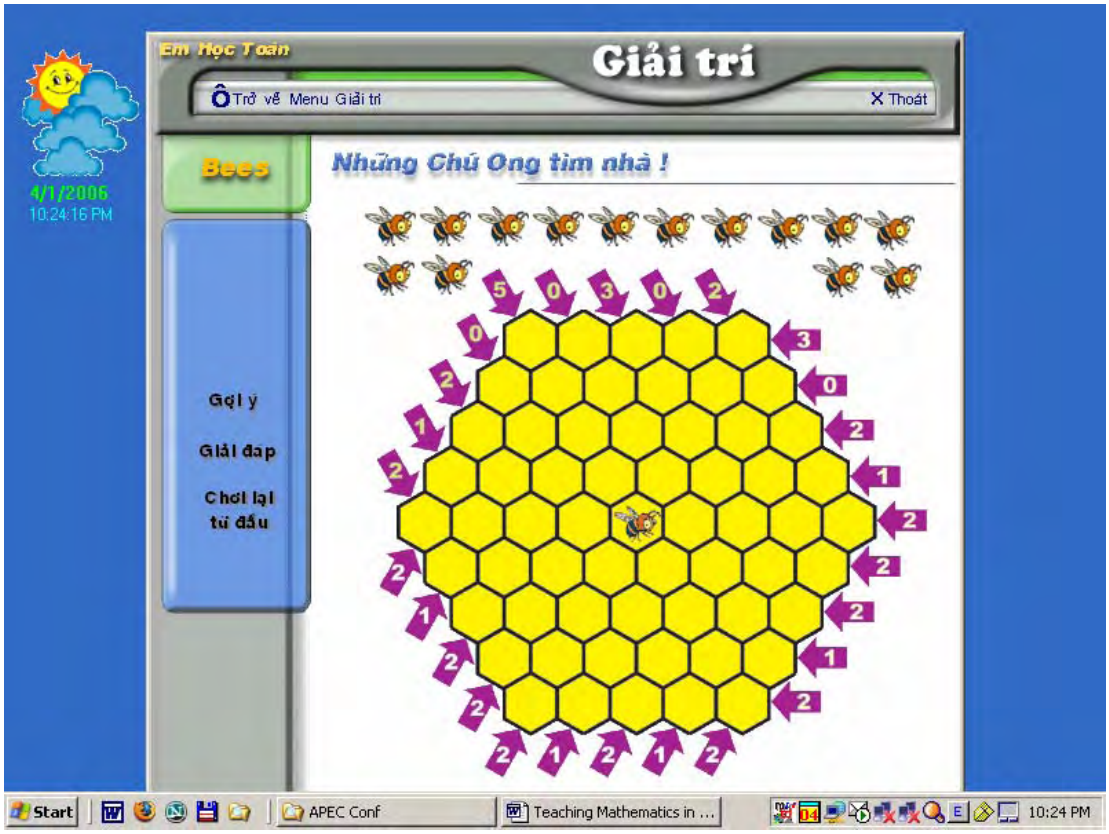


2.2. Numbers within 100



3. PART THREE: “Relax with logic thinking”

Here is an example. There are 15 bees among them only one bee could already find a “room” within the matrix of rooms. It is known that rooms of bees are not adjacent, and the number shows the quantity of bees on the line. Find the rooms for all bees.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HELPING STUDENTS DEVELOP AND EXTEND THEIR CAPACITY TO DO PURPOSEFUL MATHEMATICAL WORKS

Tran Vui

Department of Mathematics, Hue College of Education, Vietnam

The reform curriculum of mathematics at all level from grade 1 to 12 have been tried out in Vietnam. From school year 2006-2007 the new curriculum and textbooks will be implemented in the whole country. The purpose of the reform is to activate the learning mathematics of students. The curriculum tries to lessen the training of basic skills and procedures in mathematics but increases more hands-on activities to help students grasp the mathematics ideas and can apply knowledge in solving real-life problems. The mathematics teachers have learnt the effective teaching strategies by using manipulative materials to teach mathematics in problematic situations. In this article we will present some findings of our research at provincial scale in Hue city of Vietnam. A try-out lesson done by an experienced mathematics teacher will be illustrated. From our research findings and observation of the lesson we will discuss what is the good practice in mathematics education in our economy and how to implement the good practice in our new curriculum.

THE USE OF MANIPULATIVE MATERIALS IN DEVELOPING MATHEMATICAL CONCEPTS

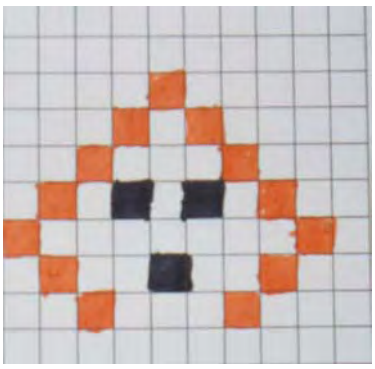
Following the old curriculum and mathematics textbooks, we taught mathematics in a traditional way. A key aim of schools was to prepare workers who were literate about numbers, computational procedures, algorithms and shapes. To ensure the memorization of basic facts, rules, and procedures, schools typically spoon-fed students, encouraging them to depend on authorities like their teachers and textbooks. If students did not know an answer of a problem, they asked the teacher or looked it up in the textbook. This is the reason why our students always need the tuition from their teachers after school. Now the new curriculum requires more than mastery of basic mathematical skills, good algorithms in solving a class of specific problems. In our increasingly complex and rapidly changing economy, the memorization of facts, rules and procedures is not enough. Business, industry, and government increasingly need workers capable of using the power of mathematics to solve new problems.

Traditional mathematics has focused on teaching concepts and skills usually in ways that are often referred to as behaviouristic. The difficulty with this approach is that mathematics becomes defined as the techniques and skills that students learn in a mechanistic way. There is seldom reference to contexts drawn from the world in which we live. Few students ever get to see how mathematics might be used and rightly question its relevance to their world. As mathematics teachers it may be helpful to

think of our role as helping students develop and extend their capacity to do purposeful and worthwhile mathematical work.

The teaching of mathematics is changing. We are challenging the old paradigm of teaching and considering a new paradigm based on theory and research that has clear applications to instruction. The teacher ought to think of teaching in terms of several principal hands-on activities. The new paradigm of teaching is to help students construct their knowledge in an active way while working cooperatively with classmates so that their talents and competencies are developed. The activity 1 below was revised from try out mathematics textbook grade 5 (page 78).

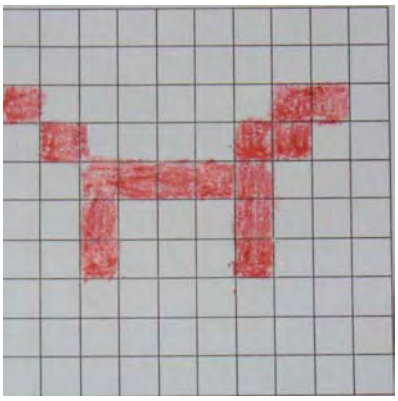
Activity 1. Use Figure 1. Student A shades any number of squares on one of the 10 10 square-grids. Student B answers the questions below the square-grid. Student A checks the answers given by Student B. Each student shades the square-grid twice and answers the questions twice.



No. of squares: _____

Fraction: _____

Percentage: _____



No. of squares: _____

Fraction: _____

Percentage: _____

Figure 1. Some student liked to draw beautiful pictures

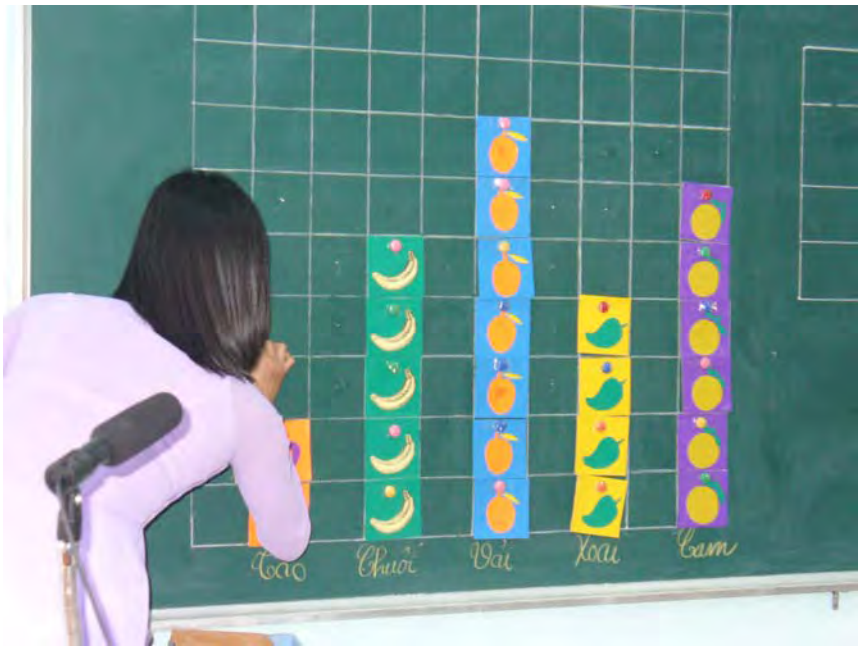
Some students liked to draw beautiful pictures such as dogs, houses, robots with the polygon shape and asked their peers to answer the above questions. They realized that the shaded region was not necessary a set of small squares but also any polygon that its area can be found easily. To do this activity most students feel that they were free to raise the questions, and knowledge of mathematics constructed was from their figures not from the teacher. This activity gave a good intuition on the relationship between fractions, decimals and percentages, students actively involved in doing their works.

DEVELOPING MATHEMATICAL CONCEPTS FROM REAL-LIFE CONTEXTS

Traditional mathematics has focused on teaching concepts and skills usually in ways, which are often referred to as behaviouristic. The difficulty with this approach is that mathematics becomes defined as the techniques and skills that students learn in a mechanistic way. There is seldom reference to contexts drawn from the world in which we live. Few students ever get to see how mathematics might be used and rightly question its relevance to their world. As mathematics classroom teachers it may be helpful to think of our role as helping students develop and extend their capacity to do purposeful and worthwhile mathematical work. The critical manifestation of mathematics power in the mathematics investigation lies in the students' abilities to employ mathematical thinking, understanding, tools, techniques and communication skills.

Activity 2. Students in a class are surveyed to find their favourite fruits. The fractions, decimals and percentages of each fruit choice are determined. These percentages are then marked on a 100cm strip of paper, where 1cm length represents 1%. The strip is then bent into the shape of a circle to make a pie chart. In this activity the students work in groups of four, they generate their own data, make graphs and analyses, and present these findings back to the whole class.

These activities are designed to reinforce the general concepts, imagination and concept of conversion from fractions to percentages.



Students record their own data in the table 1:

Fruits	Apple	Banana	Lychee	Mango	Orange	Total
No. of students						

Table 1. The number of students favouring each type of fruits.

Using these data students make a bar chart to compare the number of students favouring each type of fruits. And then students determine and write down the fractions, decimals and percentages of each fruit choice in the table 2.

Fruits	Apple	Banana	Lychee	Mango	Orange
Fractions					
Decimals					
Percentages					

Table 2. The fractions, decimals and percentages of each fruit choice.

The real-life problems are of real relevance to students, because they impinge on their everyday lives. When tackling such problems, students may draw on any knowledge, skills and understanding at their disposal, including those from outside mathematics. Real-life problems are often ill-defined, contain insufficient or redundant information, and may have several alternative solutions. When a problem is obtained, it is put into practice.

In one pilot class of 24 students, the data obtained by students as shown in the table 3.

Trái cây	Táo	Chuối	Vải	Xoài	Cam
Phân số	$\frac{2}{24}$	$\frac{5}{24}$	$\frac{7}{24}$	$\frac{4}{24}$	$\frac{6}{24}$
Số thập phân	0,08	0,21	0,29	0,17	0,25
Tỉ số phần trăm	8 %	21 %	29 %	17 %	25 %

Table 3. The data generated by students in a class of 24 students.

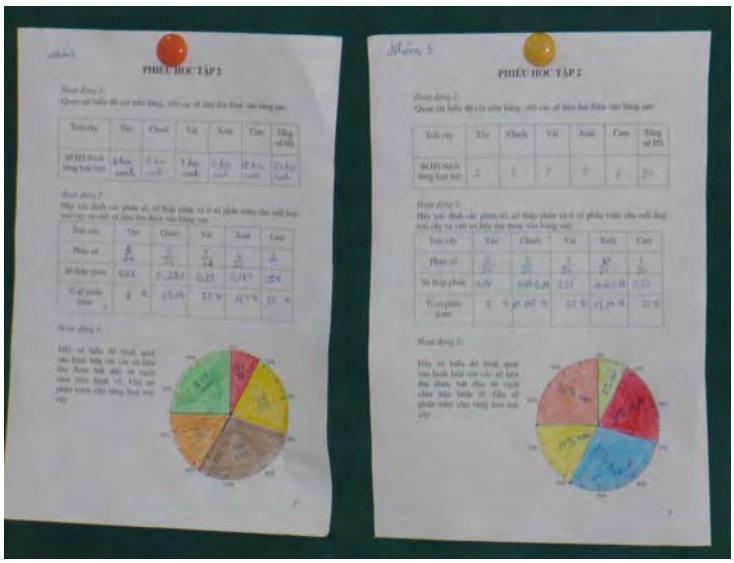
Most of students got the difficulty in deciding which decimals they should use when transfer fractions $\frac{2}{24}=0.083$, $\frac{5}{24}=0.208$, $\frac{7}{24}=0.291$, $\frac{4}{24}=0.166$ to the decimals.

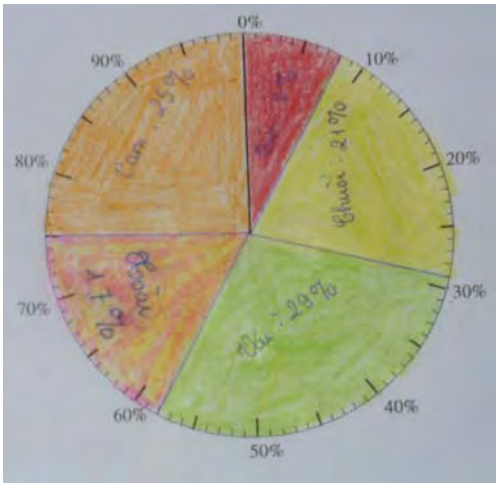
Some students asked why $\frac{4}{24}=2\times\frac{2}{24}=2\times 0.08=0.16$ but the approximate decimal was 0.17. Most of students did not check the condition 8% + 21% + 29% + 17% + 25% = 100%.

These percentages are then marked on a 100cm strip of paper, where 1cm length represents 1%. The strip is then bent into the shape of a circle to make a pie chart.

Students made their own the pie chart from the data generated.

If we totally direct the activity, the problem becomes ours not the students who will subsequently lose interest. How can we expect students to use mathematics outside school if we never give them a chance to make decisions in the classroom? The students have to be given the chance to learn from their own mistakes.





The experience of having worked on a real-life situation may motivate and enable students to perceive the value of techniques when they are introduced. Students may, however, still not able to use the techniques on their own unless they are given further opportunities to apply them in various other real investigative contexts.

USING DYNAMIC MODELS TO MAKE MATHEMATICS INTERESTING

Activity 3.

Work in pair using moving circles.

Instructions:

1) Student A rotates the moving circle to an arbitrary marked position.
2) Student B answers the questions in the figure .
3) Student A checks the answers given by Student B.
4) Repeat (1) to (3) to begin with Student B.

The students are familiar with moving circles in learning fraction as part of a whole. This time they were excited with learning percentage with the same model. Some students liked to rotate with two or three moving circles and asked their peers to answer some questions created by curious students such as what is the fraction of each part of the circle, what is the percentage and how to count them faster.

Fraction:_____Percentage: _____% Fraction:_____Percentage: _____%

Figure 2. Some students liked to rotate with two or three moving circles

The dynamic models really make mathematics interesting. This model can be presented effectively by dynamic software such as Geometer's Sketchpad.

PROBLEM SOLVING AS AN EFFECTIVE TEACHING STRATEGY

The need for problem solving work plays a vital role in our reform mathematics curriculum but how one can use this method depends much on the "teaching styles" of a teacher and the "learning styles" of the students. The students and class-room teachers are not familiar with the problem solving strategies. They still want to teach more algorithmic procedures to students to be sure their students can get high scores in the examination.

Problem 1.

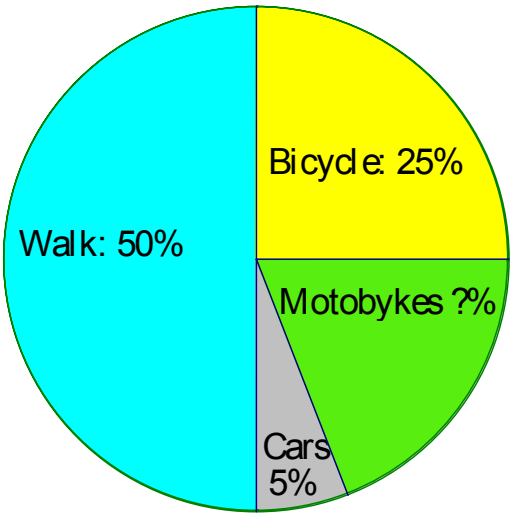
The pie chart on the right shows the means of transportation that 80 students using to go to school. In the chart the percentage of student using motorbikes is missing. Using the chart to find:

Number of students walked: _____

Number of students used bicycles: _____

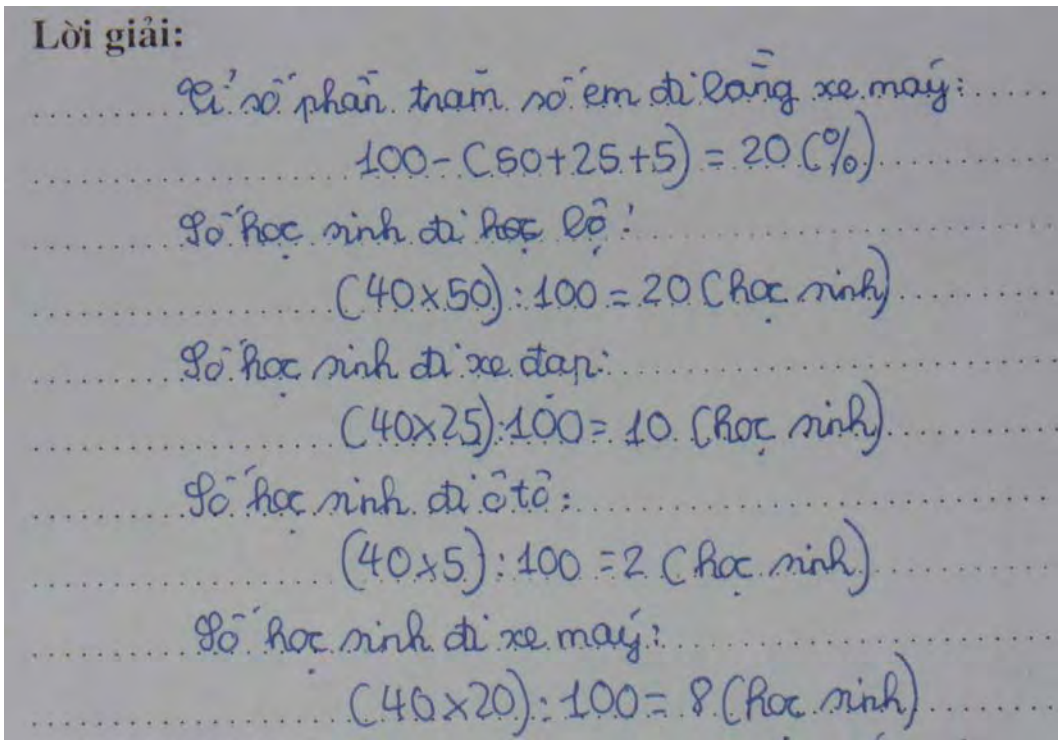
Number of students sent by motorbikes: ___

Number of students sent by cars:_____



The pie chart in problem 1 has a missing number, students felt difficult to find it. They have to guess and check their prediction and then find the method to solve the problem. At the beginning students got stuck because this is a no routine problem. They

exchange the ideas in group of four to find an effective approach to solve the problem. A solution of one group was illustrated below.


Lời giải:

Tỉ số phần trăm số em đi bằng xe máy:

100 - (50 + 25 + 5) = 20 (%)

Số học sinh đi bộ:

(40 × 50) : 100 = 20 (học sinh)

Số học sinh đi xe đạp:

(40 × 25) : 100 = 10 (học sinh)

Số học sinh đi ô tô:

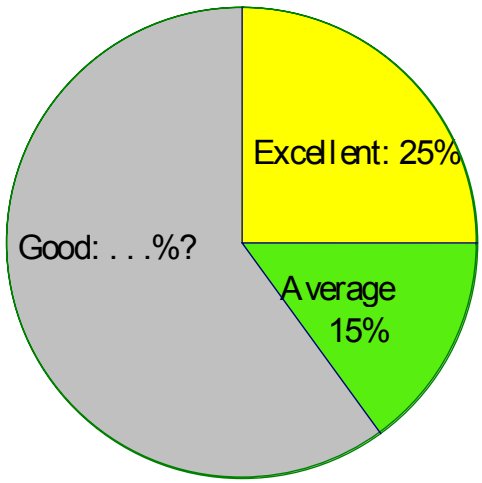
(40 × 5) : 100 = 2 (học sinh)

Số học sinh đi xe máy:

(40 × 20) : 100 = 8 (học sinh)

Problem 2.

The pie chart on the right shows the ranking of the mathematics achievement of grade 5 students in Tran Quoc Toan school. The percentage of good students is missing. If the number of good students is 120. Find the number of students with excellent achievement, average achievement.



In problem 2, students wondered why the total number of grade 5 students was not given. And then another number was missing. When they understand the meaning of percentage as learnt above, they feel confident to solve this problem.

DISCUSSION

- The aim of good practice in teaching mathematics is to help students make sense of their world by equipping them with mathematical skills. These skills include content skills or "what mathematicians know" and process skills or "what

mathematicians do."

- The good practice should balance the content skills and process skills including the problem solving process. These two types of skills are always necessary to students in solving new problems in their lives.
- This emphasis on the processes of mathematics, with problem solving being the core, is evident in our new curriculum initiatives. Some teachers, wary of change and concerned about "jumping on bandwagons" have ignored these processes, and continue to teach only the content of mathematics and algorithmic procedures.
- When the teachers use manipulative materials in teaching mathematics they recognized that their students more active in learning. The students liked to learn mathematics with dynamic or moving models.
- The teachers have to learn how to create new mathematical models with problematic situations and prepare good manipulative materials.
- Students can often generate their own activities and questions. However, it is helpful if we prepare a sheet with a list written out of possible ideas as a resource for students who genuinely cannot see any possibilities for them to reflect on.

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GOOD MATHEMATICS TEACHING PRACTICES - IN THE MAKING: A PHILIPPINE EXPERIENCE

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This paper discusses how an eighth grade mathematics teacher engaged his students in hands-on activities, made them work on tasks in small groups, encouraged them to solve problems in different ways, and seized their mistakes as learning opportunities. Although the way he carried out these teaching practices still requires much improvement, these enabled the students to make sense of mathematics by discovering mathematical relationships on their own, communicating their ideas and reasoning logically, exploring various ways of solving problems, and regarding making mistakes as part of learning. These experiences put the students as the most important factor in the teaching-learning process. Such is a big departure from the traditional classroom scenario where the teacher is the source of all learning.

A CONTEXT FOR GOOD MATHEMATICS TEACHING PRACTICES

In order to understand what good mathematics teaching practices mean in relation to the class described in this paper, it is necessary to know the characteristics of mathematics classes in the Philippines generally, the recommendations of the Department of Education on strategies in teaching mathematics, and the perceptions of key mathematics teachers regarding effective teaching strategies. Likewise, it is important to know the methodology used to gather data that captured these good practices.

What is a Mathematics Class in the Philippines Like in General?

To a great extent, the teacher explains and asks questions in a whole class setting. If group work is done, it is superficial. When students discuss, they seldom can sustain the discussion and make it productive (Pascua, 1993). Students are orderly and quiet. To begin a new topic, the teacher first asks students what they know about it then explains the definition and rules (Department of Education, et al 2000). The most common strategies in teaching mathematics are exposition, practice and consolidation, and discussion (High School Mathematics Education Group 1996; Bernardo, Salazar-Clemena, and Prudente 2000).

Department of Education's Recommendations and Key Teachers' Perceptions

The 2002 Basic Education Curriculum in Mathematics in the Secondary Level which is currently being implemented advocates using a variety of teaching strategies among which are practical work, discussion, problem solving, investigations besides

exposition and practice and consolidation, as well as cooperative learning (Department of Education 2002).

The teaching strategies perceived to be most effective by science and mathematics teachers of schools identified as benchmarks in teaching and learning practices were: hands-on experience that brings students to their fullest learning capacity because they depend on themselves, cooperative learning because they can share better knowledge when they work in groups rather than when they work alone, and self-discovery because it enhances students' learning capability (Penano-Ho 2004).

How were the teaching practices documented?

The source of data in this paper is the 21st section of a grade 8 mathematics class consisting of 57 students in a public secondary school in Metro Manila. It was one of the three Philippine schools included in the international research Learner's Perspective Study which focused on the teaching and learning process that went on in grade 8 mathematics classes taught by locally identified competent teachers. The class was observed and videotaped for 15 consecutive school days with the first 5 days serving as familiarization period. Three cameras were used: one focused on the teacher, another on the whole class, and still another on two focus students who were randomly selected daily. There was on-site mixing of the teacher and focus students' cameras. A microphone placed between these students picked up their conversations. At the end of each class that lasted on the average for one hour, the focus students were interviewed one after the other. The teacher was also interviewed at the end of each week. The video-stimulated interviews were audio-taped and along with the mix videotapes transcribed. Translation was done when needed because although English is the medium of mathematics instruction, both the students and the teacher at times code-switched to Filipino, the national language. This paper used the data from the mix videotapes, teacher interview, and lesson plans for the last 9 days. The lessons were on geometry, particularly conditions for right triangle congruence, quadrilaterals and their properties, and different kinds of parallelograms and their properties.

GOOD TEACHING PRACTICES

A typical mathematics class usually employs the question and answer type of exposition, the teacher starts with definitions and rules, and students are most of the time quiet and just listen to the teacher. In contrast, the teacher in this study used hands-on activities for practical work to introduce a topic and elicited discussion among students when they worked in small groups, presented their various ways of solving problems, and corrected their mistakes.

Using Hands-on Activities

In 2 of the 9 lessons, the teacher used practical work besides exposition. In lesson 8, instead of giving the definition of a median and altitude that will be used in a subsequent construction of a proof, he asked the odd-numbered groups to draw any triangle and a segment from any vertex to the midpoint of the opposite side. He also

asked the even-numbered groups to draw any triangle and a segment from any vertex perpendicular to the opposite side. This was an instance where not all the students were doing the same task and individually within a group, students were free to choose what kind of triangle they will consider. Later, the teacher asked the students to analyse their work and compare it with their seatmates. If only the teacher did not add that the students may draw an acute, right, or obtuse triangle and left the students to think about this on their own, this could have been an open exploration activity. After a sample answer from each task was presented, the teacher brought out the term median and altitude, the important words that he would use in the lesson. So the activity provided a context in which these two words were introduced and its additional value was that students observed that regardless of the kind of triangles that they drew, the three medians and the three altitudes always intersect at a point and that in fact, a triangle can have three medians and three altitudes or heights. According to the teacher, students usually encounter the word altitude or more familiarly the word height only in formulas. And so here, he wanted them to realize that this is the same height that is involved in proofs. So he tried to make connections from what they had known in measurement to what they were learning in geometric proofs.

In Lesson 11, the teacher used practical work to make the students verify the following relations after they had established the proofs: in a parallelogram, opposite sides are congruent, consecutive angles are supplementary, and opposite angles are congruent. He asked some students to use the blackboard meter stick and blackboard protractor to measure the sides and angles, respectively of his drawing of a parallelogram. Thus, the students had the opportunity to confirm that what holds true for the general case also holds true for a specific case.

Using practical work is a good teaching practice because it enables students to discover on their own abstract relationships through concrete means. By using this, students will take on greater responsibility for their own learning rather than merely rely on the teacher (National Council of Teachers of Mathematics 1989). By providing students with appropriate activities and facilitating the processing of the results, this can be achieved. Perhaps, would-be teachers in teacher education institutions can be taught how to develop such activities especially for topics that students find difficult to learn. They can also be taught how to conduct action research to determine if the strategy really helps in better student learning.

Using Groupwork

A dominant feature of the lessons, that is in 7 out of the 9 lessons, was the use of group work. Since students were organized by tables, those seated around a table consisted one group. Some groups had 9 members while others have 10. In lessons 6 to 8, group work involved performing exercises on making proofs. In lesson 9, there was a test and in lesson 13, the teacher carried out a whole class discussion on the different properties of parallelograms, so in both lessons there was no group work. In lessons 10, 11, 12, and 14, group work involved exercises on computations.

Working in small groups afforded students opportunities to ask questions intended to get help or clarify their thoughts and to communicate their ideas clearly and reason out logically so that they could be understood whether they were asking or answering questions or simply discussing their ideas.. An example is in lesson 12. The group of Arn and Sher was asked to determine the measures of all the angles of parallelogram CITY given that angle C is equal to 5x – 10 and angle T is equal to 4x + 10. Sher wrote their answers on the manila paper which the teacher gave. Following are the conversations of Sher and Arn.

Arn:	How did that happen? Why are I and Y 90?
Arn:	What's this consecutive? Who's going to explain?
Sher:	Arn
Arn:	Consecutive? Consecutive angles?
Rub:	Angle I and angle T?
Arn:	What's that? Supplementary? Are these supplementary? Supplementary?
Arn:	What?
Sher:	What? Which? Which is your problem here?
Arn:	I'm asking if these are supplementary?
Sher:	Supplementary? They are equal because aren't C and T opposite angles?
Arn:	Yes.
Sher:	Opposite angles are congruent, oh. So it's written there, angle C is equal to angle T. 5x minus 10 is equal to 4x plus 10.
Arn:	Yes.
Sher:	There, then just find their value.
Arn:	Okay. Why does it not have this?
Sher:	What? It's there already, oh.
Arn:	What I mean is, why is it like that? … Why?
Sher:	There's no more like that because 5x minus 4x is already x. You don't need to get it.
Arn:	Wait, wait.
Sher:	4x. 5x minus 4x is equal to 1x, eh we divide don't we?
Arn:	5x
Sher:	Minus 4x.
Arn:	Oh, 1x.
Sher:	So do you still need to divide 1x by 1?
Sher:	Isn't it that you don't need to? Isn't it that it's just the same as x?
Arn:	Sher…Sher…. Psst, Sher. Eh, what is, what is angle I? Consecutive angles?
Sher:	Yes, because C and angle what, angle C and angle I are consecutive angles.

Sher: It means that they are supplementary, their measure is 180.

Arn: Whatever is the measure should they be 180?

Apparently, Arn did not stop asking Sher until he got satisfactory answers to all his questions. Sher patiently explained because Arn would present the work of their group for others had their turn already. In fact, for a while Arn asked Sher to present in behalf of their group because she knew how to do it. Nonetheless, because by asking questions, Arn came to understand their solution, he presented it. This was the kind of support that group members got since the atmosphere was one of cooperation rather than competition. During group work, the teacher went around the room to monitor and assist the different groups. Arn did not have to wait for the teacher to come and help him because somebody in his group was already capable of doing that.

As it was, small group work had already its merits but these were limited. For instance, the group size was too large to fully involve every member in the discussion of the solution. There should had been two smaller groups per table instead of just one. The best student in each group almost single-handedly thought of the solution while the others simply looked on and asked him or her or other group members, if at all, when they did not understand something. The most insistent ones in asking questions were those who would present the group's output. Some groups finished quickly, and so were off-task and sometimes noisy while the other groups were still working. If only the teacher were sensitive to these situations, then he could have converted group work to cooperative learning whose distinctive features are complete participation and individual accountability for knowing what was done (Johnson and Johnson 1990). The benefits of cooperative learning aside from the development of communication skills such as better mathematics achievement and positive interpersonal relationships are well-documented in research studies (Webb 1991, Fitzgerald and Bouck 1993, Pickhard and Bingaman 1993). Hence, if properly implemented as a cooperative learning group, using small group work is a good teaching practice. In the Philippines where classes are big and so the teacher is not always readily accessible for help and where resources are limited, cooperative learning offers peer help and resources sharing.

Encouraging multiple solutions to problems

The teacher also gave the students the opportunity to work on the tasks he assigned the way they decided to. For instance, in Lesson 11, the group of Nic and Jean were asked to find the measure of each angle of a parallelogram MORE given that the measure of angle R is 5x and the measure of angle E is 4x. At the time that they were working on this routine problem, the teacher approached them. The teacher asked them what the relation between angle E and angle R was to which Nic correctly responded "supplementary". When the teacher probed for the reason, members of the group also correctly answered "consecutive." The teacher then told them where to write their solution apparently thinking that they would use the relation that consecutive angles of a parallelogram are supplementary, to get the value of x. That is, $5x + 4x = 180$. So $x = 20$. On the contrary, Nic said: "Given if measure of angle E

is equal to 4x. Measure of angle R is 5x. Eh, then … this. This is 5x, and 4x also (referring respectively to angles M and O)." He made use of the relation that the opposite angles of a parallelogram are congruent and that the sum of the measures of the angles of a quadrilateral is equal to 360. He got 5x + 4x = 9x. 9x + 9x = 18x. Then 360/ 18x = 20. Substituting the value of x for the measure of angles E and R, he got the measure of each of the angles. Nic later presented their work. After he had presented, the teacher made the following comments.

Teacher: What can you say about the solution of this group and this group? Is there any difference?

Student: Yes.

Teacher: Yes. But you got the same answer, is that right? Let's see. What did you use here Mike? This is your answer Mike. What did you use here? The? Yes, Mar.

Mar: The…the what. Equal measures.

Teacher: Measures…Here, here. Angle A and, ah, is equal to 5x, 7x. So 5 plus 7 is equal to 12x. That is? What did you use? …Mike.

Mike: Consecutive angles.

Teacher: Yes, consecutive angles. So what about what about Nic?… You use here?

Nic: The sum of the quadrilaterals.

Teacher: Okay, the sum of the measure of angles of a quadrilateral. Here he used many. Angle E and angle O. You know these are opposite angles, aren't they? So 4x, 4x, 5x, 5x…Although this is quite long, but this is correct. And, and I encourage you to … ah, to use that kind of behavior. Because if you really cannot think like that immediately, eh then try another what try another method, right? O another way … in finding the correct answer. O last group?

Later during the quiz, the teacher told the class to use relations so that they could cut down on computations. This shows that while he accommodated the long but correct solution of students he at the same time was quick to point out that there was a more efficient way. This balance between accommodating student responses that may differ from what a teacher expects and making them realize that some ways are better than the others, is a good teaching practice. On one hand, students will get the impression that they are capable of coming up with their own solution to a problem no matter how crude or less elegant it may look and this can build their confidence. On the other hand, it can create an inclination in them to explore other solutions (NCTM 1989). Moreover, given different solutions, they can compare their merits and evaluate which one may be better than the others and identify the reasons for their choice.

Viewing Mistakes as Learning Possibilities

Students' group work output at times had mistakes. For example, in lesson 6, one group made an incorrect statement in their proof. When the teacher was already discussing their proof, and he probed the students concerned about what they meant

by the statement, Win said: "Sir we just made a mistake in that statement." Emy who wrote their proof admitted that she really made a mistake.

Teacher: Do you realize your mistake?

Emy: Yes. That's already correct.

Teacher: That's alright with me as long as you see…You don't repeat the same mistake. We just keep repeating. Oh, here do you need to bisect?

Another example is in lesson 14. After Mar presented the work of their group with the class following his presentation and the teacher even writing the numbers that he said, Sher commented on his work.

Sher: Sir angle 1 and angle 2 are not supplementary. They are congruent. {Students cheer.}

Sher: Because what you (referring to Mar) did was to add the two and equate to 180 which should not be.

Teacher: Oh that's right! {Teacher analysed the problem. Students were noisy.}

Mar: Sir all are already wrong.

Class: That was embarrassing! {Students teased Mar.}

Teacher: Yes that is correct. That's okay.

Mar: Sir how?

Teacher: No, no. That's okay.

Class: That's okay. That's okay. That's okay, Mar.

Teacher: That's okay. So angle 1 and angle 2 are really not supplementary. But they are congruent. … Please do them all again. The last part only. Anyway that is very easy than the other one. Alright, just sit down again there. …Okay next group, next group.

Mar solved again by himself on his seat. A group mate said that he was pitiful because he was solving alone. His group mates encouraged him by saying that he could do it. When he presented for the second time around, his answer was already correct.

In both lessons 6 and 14, the teacher said that it was alright if students made mistakes. Moreover, he gave them the chance to correct their mistakes thereby turning the situation as a learning opportunity. With the kind of accommodating atmosphere that this manner of responding to students' incorrect responses foster, students would more likely not hesitate nor be afraid to solve problems. Yet at the same time, the teacher admonished the class not to be careless in their responses. For instance in lesson 6, she told the groups: "Do not just write for you have nothing more on which to write. You should think. Think. You should think first." It was observed that only a small sheet of manila paper was provided to each group on which to write their solution. Apparently, the teacher wanted the students to strike a balance between taking risks that might entail committing mistakes and not being careless. His reassuring words and also that of the class exemplified a learning environment referred to by (Boland 1999) where it is safe to take risks because trust has been

developed and risk taking and sharing are valued. As such the way he dealt with erroneous responses of students is a good teaching practice.

WHAT ELSE NEEDS TO BE DONE

While the good teaching practices identified in this paper may still be greatly improved, they are definite attempts to implement the recommendations contained in the curriculum on how mathematics should be taught. They are also in line with what the local key mathematics teachers regard as effective teaching practices which in turn are attuned to the thrusts of the international mathematics community. In particular, the teacher will have to be familiarized more on developing and giving open-ended activities involving practical work. He should be coached on how to form and manage cooperative learning groups to maximize the benefits that may be derived from using them. Both of these may be done by mentoring where a more experienced and knowledgeable teacher serves as the mentor. Thus, at present these good teaching practices are still evolving or in the making.

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Changes in primary school math classroom since the new curriculum reform

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Since "Mathematics Curriculum Standards in the Phase of Full-time Compulsory Education (Experimental Manuscript)" (referred to as the Standards) was enacted six years ago, the new math curriculum reform has been carried out all around China. Teachers tried to integrate the ideas and requirements of the Standards into their daily teaching practice, which brought new look to math classroom. The ideas, objectives, contents and suggestions that the Standards involved have increasingly become the focus of attention and the base of research for the mathematics educators and researchers. In light of this context, a series of obvious and profound changes have taken place in the primary school math classroom instruction.

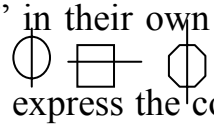
1. The focus now is placed on the close association of math and daily life, and the students are encouraged to learn math in rich contexts.

The math classroom instruction has changed from the former teaching mode of reviewing——introducing the new knowledge——giving examples——giving exercises. The math classes have become more interesting and lively. Teachers create rich contexts such as fairy tales and problems in daily life to attract and guide students to enter the mathematical world.

The new curriculum advocates that the contexts should be close to students' daily life, with the aim to enable the students to feel the close association of math and life and thus feel the need and pleasure to learn math. So math instruction should start from students' previous life experiences and personal knowledge so that the students can experience the process of abstracting the practical problems in life into mathematical models and then explain and apply these models. Consequently, how to create interesting contexts that are close to life, full of mathematical implications and need exploration has become the focus of teachers' research.

As a matter of fact, students learn math long before they go to school and beyond the math classroom. In daily life they often run into various kinds of mathematical problems and have formed some kind of personal math knowledge. Though informal and unsystematic, and sometimes obscure and unclear, and some even wrong, these knowledge and experience are the starting point for students' further math learning.

Here an example in math instruction will be given to illustrate this. In one math lesson, the objective was to help the students have initial knowledge of fractions. First, the teacher wanted the students to learn the mathematical concept of 1/2. The teacher started from the question of how to divide one apple into two equal parts to

two children. From their previous life experience students answered that each child should get “a half”. Here the teacher didn’t immediately write 1/2 on the blackboard, instead, he asked the students to express the concept of “a half” in their own ways. Most of the students employed graphs to show this (such as ), but one student used half of one of the characters in his name to express the concept of “a half”. The teacher now didn’t eagerly make comments on their various expressions, but introduced the mathematical way of expressing “a half”, i.e., 1/2. He then asked the students whether they liked this expression. Some students still thought that their own ways were better because anyway, graphs are more vivid than figures. Then the teacher asked them to express “one a hundredth” in their own way. Now all the students realized that the mathematical expression of “1/2” was easier and can be generalized, so they accepted this new mathematical concept gladly. In this example, the teacher not only provided opportunities for the students to show their own expressions, but also brought the students to a full understanding of the new mathematical concept through elaborately designed questions. In other cases, some teachers encouraged students to create a mathematical way to express the concept of “a half”. So the students created 1/2, 2/1, 2, and 2^1, all of which were not only the students’ innovation, but also their primary understanding of fractions.

It’s very natural that children have an immature primary understanding of certain mathematical questions. This crude understanding is most individual and is the real reflection of math in children’s mind. It is right on the basis of this incomplete and inaccurate expression of math that children will come to a real understanding and a correct expression of math.

2. Students’ learning styles are more diversified, and inquiry learning, collaboration and communication are becoming important math learning styles.

In the current math curriculum reform in primary schools in China, an important idea is to encourage various learning styles. As a result, students’ independent thinking, practical abilities, collaboration and communication between teachers and students and within students themselves are becoming important learning styles. Learning is becoming an active, interesting and highly individualized process of thinking and practicing. The changes in these aspects can be seen from the following two examples.

In the past, when teaching arithmetic, great efforts were made in training students to “calculate fast and correct”. In terms of arithmetical methods, the teacher focused on explaining one particular method and making sure that all the students master this method through large amounts of various exercises and drills. For example, to solve the problem of 15-9, traditional math instruction requires the students to use the method of “to solve subtraction by addition”, i.e., 9+6=15, so 15-9=6. But with the further implementation of mathematics curriculum reform in primary school, teachers come to realize that when students try to do arithmetic, they have their own different arithmetical methods resulting from their own particular life experience and

individual ways of thinking. The students can and should invent their own arithmetical strategies, which will be of great help to their understanding of mathematics. Meanwhile, all the students can benefit from listening to and responding to others' methods. In addition, students' strategies show their ways and levels of thinking, which helps teachers to reflect on and improve their teaching. So, in math instruction, teachers should encourage and respect students' independent thinking, and provide opportunities for students to share their different arithmetical methods. In the current math classroom teaching, teachers usually ask the students to think out various ways of solving 15-9, and then share with the whole class. Some students subtracted 9 from 15 one by one. Some divided 15 into 10 and 5, 10-9=1, so 1+5=6. Some divided 9 into 5 and 4, 15-5=10，so 10-4=6. Some thought that 9+6=15, so 15-9=6. Still others thought that 15-10=5, so 15-9=6. For these different methods, the teacher gave positive feedback and encouraged students to share their methods with one another. Through communication the students finally chose the best method that fit them. Here students are no longer required to use the same single method as before.

In the reform of classroom teaching, teachers come to realize that putting forward a problem and solving the problem are equally important. But putting forward a problem is the weak point for Chinese students. So some teachers try to provide opportunities for students to put forward a problem. They set a relaxing environment and encourage the students to observe life from various perspectives, describe things and phenomena from mathematical perspective, discover the elements in them that are relevant to math and put forward a problem, no matter correct and mature or not. The following is an example of a classroom teaching of "basic knowledge of percentage". At the beginning of the class, the teacher asked the students to share examples in life of "percentage". They found many on the package boxes of beverage, on the labels of clothes, on newspapers and on the instructions of toys and became greatly interested in it. Then, encouraged by the teacher, the students asked questions about percentage from different perspectives. The following are several typical ones:

(1) Why do people like to use percentage?

(2) What's the difference between percentage and fractions?

(3) What does percentage mean?

(4) How to write percent in mathematics?

(5) What's the use of percentage?

(6) Which is more widely used, fractions or percentage?

After summing up these questions, the students tried to solve them in groups. And then the teacher and the students summed up what they had learned about percentage. At the end of this lesson, the teacher again asked the students to ask questions. The students talked actively. Some of them asked whether there was a way of describing an amount as if it was part of a whole which was 10 or 1000. This not only promoted

the students to have a more profound knowledge of what they'd learned, but also encouraged them to innovate.

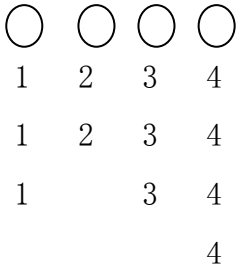
3. The teacher creates a democratic and relaxing classroom atmosphere, where the students have more opportunities to innovate

In the classroom reform, the teachers come to the common recognition that math classroom is the basic place where students create, share and communicate. So the teachers all try their best to provide a relaxing classroom atmosphere, and provide more opportunities for the students to share their ideas and strategies. In this way different ideas collide to produce illumination to each student and thus promote the common development of different individuals.

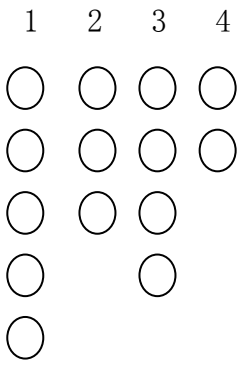
The following is an example of the lesson "statistics" for students of first grade. The students were divided into groups and each group had a bag with four balls marked No. 1 to No. 4. The group members were asked to keep a record of the number of the ball each time they take blindly from the bag. Then the teacher asked them to share how they made the record.

Group one: We wrote down the number of the ball each time we took it from the bag. Like this: 4 1 1 2 3 4 2 1 2 3……

Group two: We first drew four circles representing the balls of No. 1 to No. 4, and then we wrote down the number of the ball each time we took under its corresponding circles. Like this:



Group three: Our practice is a little bit different from group two. We first wrote down No. 1 to No. 4 to represent the four kinds of balls and then we drew circles under them each time we took a ball. Like this:



……

And then the teacher asked the students to make comments on the above mentioned methods:

Student 1: I think all of them make good records.

Student 2: I disagree. The record of group one is not clear. Both group two and group three are better.

Student 3: I also think that the recording methods of group two and group three are better than that of group one because the results are quite clear.

Student 4: I think the method of group three is the best because it's very easy to just draw a circle.

From the discussion we can feel that through equal communication, the teacher guided the students to share their ideas and thus formed an enthusiastic and orderly learning atmosphere. The teacher listened, asked questions and guided the students to share their ideas and products. Even for those "not so good" ideas, the teacher didn't simply correct the mistakes but encouraged the students to discuss and make their own judgment. The teacher now not only focuses the mathematical knowledge itself, but also shows great concern for the promotion of mutual understanding, respect and appreciation between students during the process of discussion.

4. The teachers' professional development is achieved in the process of promoting the all-round development of the students

In the new curriculum reform, the idea of "to promote the all-round development of each student" has been fully carried out in daily teaching. According to the two years of follow-up research and evaluation of the first round of 42 national level experimental regions of curriculum reform conducted by Ministry of Education basic education curriculum reform "professional supportive work group", there have been obvious improvements in students of the experimental classes. The students now enjoy learning more and like going to school. There have been improvements in students' all-round quality, ability to search and process information, communication and expression skills, questioning and innovative abilities as well as practical abilities. All these strongly show that great improvements have been made in primary school math classroom reform.

At the same time, the implementation of the new curriculum also forcefully promoted the teachers' professional development and facilitated the changes in teachers' roles. The following are the new roles of teachers:

1. Teachers as facilitators of students' learning.

By this it means that teachers not only transfer knowledge to the students in the traditional sense, but also promote the sound and harmonious development of he individual student stressing on their learning capabilities. The role of the teachers as facilitators of students' learning is the most obvious and direct characteristic of teachers' roles. The new curriculum promotes teachers to become the organizers, guiders and partners in the teaching and learning activities.

The role of teachers in classroom can directly influence teachers' teaching behaviors. In the new curriculum, the following changes have appeared in teachers' teaching behavior: teachers now offer guidance for the students to form good learning habits and master learning strategies, create rich teaching environment, stimulate students' learning motives, foster students' interests in learning, provide various conveniences for the students' learning, establish an acceptive, supportive and tolerant classroom atmosphere, share with the students some of their own feelings and ideas as participants in learning, seek truth together with the students and be responsible for their own mistakes and faults.

2. Teachers as researchers of education and teaching

The new ideas, methods involved in the new curriculum and the various problems that appeared in the implementation of the new curriculum can hardly be explained and dealt with by drawing on the past experience and theories. The teachers should not wait for someone else to tell them how to react and then employ others' research results into their own teaching without giving it a second thought. In this regard the new curriculum promoted the transformation of the idea of "teachers as researchers" into practice.

As a matter of fact, during the years of implementation of the new curriculum, the teachers in the experimental regions can actively undertake teaching research work with great initiative. Many teachers can now look at the various problems in their teaching practice from researchers' perspective, reflect on their own behavior, explore the newly emerged problems, and sum up their experience. Teaching and researching become complementary to each other. Many teachers now write teaching diaries. They often get together to discuss problems emerged in teaching experiments. Such communication and discussion help to solve the problems closely pertinent to teachers, through which teachers really experience the joy of doing research. This "action research" which integrates teaching with researching is the basis of teachers' further improvement, is the key for improving teaching standard and is the guarantee of the innovative implementation of the new curriculum.

3. Teachers as people who construct the curriculum

The new curriculum changed the role of teachers as people who implement to people who construct the curriculum. This is first shown in the innovative employment of the textbooks. The teachers now try to use the textbooks as curriculum resources and use them appropriately in accordance with the actual needs of their students.

Secondly, this is also shown in the development of the curriculum resources. One of the characteristics of this round of curriculum reform is to emphasize on the status and role of curriculum resources. Though there's a lack in curriculum resources, the teachers try actively to develop all kinds of resources by themselves. Practice has shown that the development and employment of curriculum resources have enlarged teachers' eyes, expanded the educational contents, enriched the experiences and life of both teachers and students, and most importantly, promoted teachers' abilities of

curriculum construction and their educational and teaching wisdom.

In one word, more concrete and real changes have taken place in the students, the classroom and the teachers since the new curriculum reform.

Study Lessons
in
Attached Schools
of the University of Tsukuba

Related documents

APEC Specialists Meeting

Mathematics Public Lesson Grade 4 Mathematics Instruction Plan

Teacher: Tsubota, Kozo, Vice-Principal, Tsukuba Fuzoku Elementary School

Research Theme:
Examining instruction that focuses on "viewing a number in relationship to other numbers, such as a product of other numbers."

1. Title Prime and composite numbers

2. About research theme

(1) Fostering rich sense of numbers

The current (2000) revision of the National Course of Study (2000) stresses that the goal of "fostering rich sense of numbers, quantities and geometric figures" is to be considered carefully. Since multiplication is introduced in Grade 2, a specific goal, "to view numbers as products of other numbers," has been included. However, this is only one specific instance of developing "number sense" that must be addressed all the way though upper elementary school. Therefore, we must constantly address number sense intentionally. Today's lesson proposes the treatment of numbers sense using the topic of "prime and composite numbers."

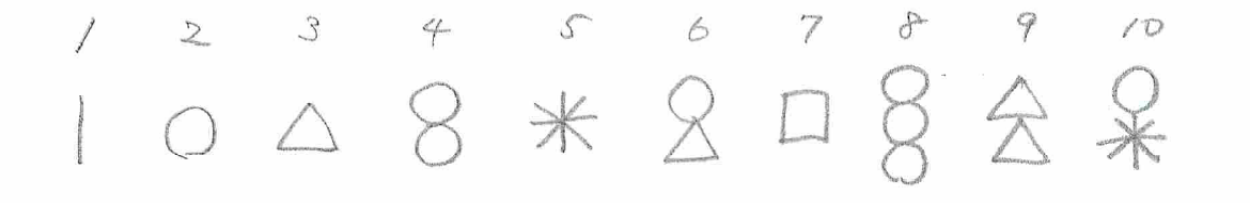
On p. 75 of Commentary on the Elementary School Mathematics Course of Study, you see a statement, "the goal is to develop an understanding of the multiplicative structure of numbers through an activity of counting objects by grouping." Within the context of the introductory treatment of multiplication in Grade 2, this statement means that students should understand that a number can be viewed as a product of other numbers. For example, 12 can be thought of as 2×6 or 3×4.

In today's lesson, we would like to further this perspective so that students can consider, for example, 12 as $2 \times 3 \times 3$.

(2) Prime and composite numbers

In this lesson, we will pictorially represent the fact that all whole numbers are either prime numbers or composite numbers, which are products of prime numbers.

The following designs will be shown, and students are expected to identify rules the govern them. Then, using those rules, students will be developing designs for larger numbers.



If students truly understand the ideas behind this lesson, they are more likely to understand the meanings of "least common multiples" and "greatest common divisors" to be studied in Grade 6.

3. Goals
To be able to view a number as a product of other numbers.

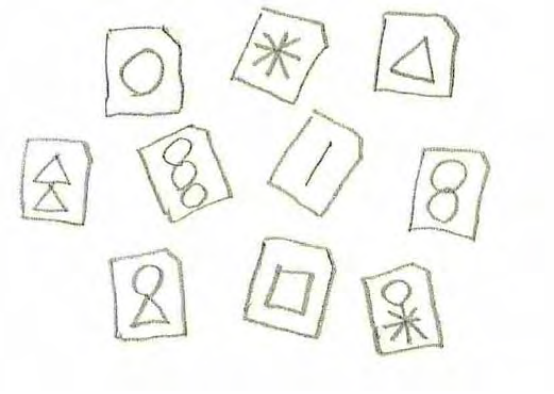
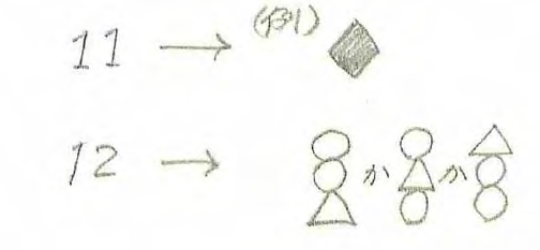
4. Instruction plan (2 lessons total)
Understanding prime and composite numbers …… 1 lesson (this lesson)
prime and composite numbers up to 100 …… 1 lesson

5. Instruction of the lesson

(1) Goals

To notice that whole numbers are made up of prime numbers and their products.

(2) Flow of the lesson

Instructional Activity	Points of Considerations
1. Observe the ten designs shown on cards and determine what they represent. 	(1) Post the ten cards on the blackboard at random. Ask students what they notice. ● If an idea that relates to numbers is raised, ask for the reasons.
2. Order the cards and identify "rules." 	(2) Guide the students to look at how the 6th design is composed. 
3. Using the discovered "rules," think how 11 and 12 can be represented. 	(3) Confirm that these designs represent numbers, then have them think about other numbers. ● Discuss and check the ideas for 11 and 12. ● Confirm that 11 must be represented by a new design while 12 can be represented by combining 2, 2, and 3. (4) Using the pattern they discovered, have students make the designs up to 20.
4. Make a chart of number designs up to 20.	

Mathematics Public Lesson Grade 5 Mathematics Instruction Plan

Classroom: Tsukuba Fuzoku Elementary School, Grade 5, Section 4
Teacher: Hosomizu, Yasuhiro, Tsukuba Fuzoku Elementary School

Research Theme:
Examining instruction that will promote the enjoyment of discovering the dependency relationship between the diameter and circumference of circles, and appreciate the beauty of the fact that the ratio of circumference to the diameter is constant.

1. Title Circles

2. About Research Theme

In a typical lesson to discover the ratio of circumference, the lesson often starts with questions like, “Let’s determine the relationship between the diameter and the circumference,” or “Let’s investigate the values of Circumference ÷ Diameter.”

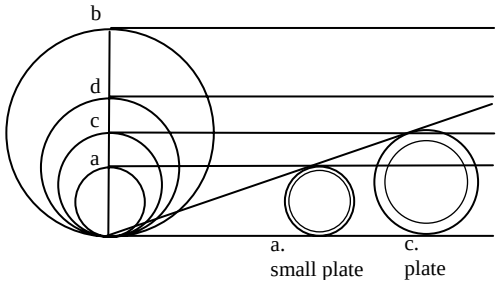
However, these questions are telling students that there is a relationship between the diameter and the circumference from the beginning. Thus, it is not quite clear what it is that children should gain from such study. Moreover, children cannot develop their ability to investigate a relationship. In particular, if the lesson starts with, “Let’s investigate the values of Circumference ÷ Diameter,” it can be legitimately criticized as a lesson that is only about practicing computation or the use of calculators.

As students discover that the circumference depends on the diameter and the ratio of the circumference to the diameter is constant, their ability to observe and think algebraically will be fostered. Therefore, in planning today’s lesson, the following perspectives were taken so that the lesson may indeed develop in such a manner that students will make these discoveries on their own.

(1) Help students realize that the circumference depends on the diameter

Roll various circular objects on the ledge of the classroom blackboard and have them predict how far they will go. By alternating large and small objects, the relationship between the circumference and the diameter become more visible.

Furthermore, to make the relationship more easily observable, organize the results in a diagram as shown on the right. The diagram will make it easier for some students to capture the relationship between the circumference and the diameter more visually.



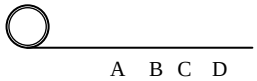
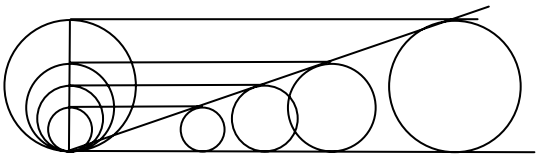
(2) Help students realize that the ratio of the circumference to the diameter is constant

In this lesson, the focus is not just have students remember the value of the ratio of circumference is approximately 3.14. Rather, by rotating familiar circular objects and measuring their diameters and circumferences, they will experience the process of gradually approaching the approximate value of 3.14.

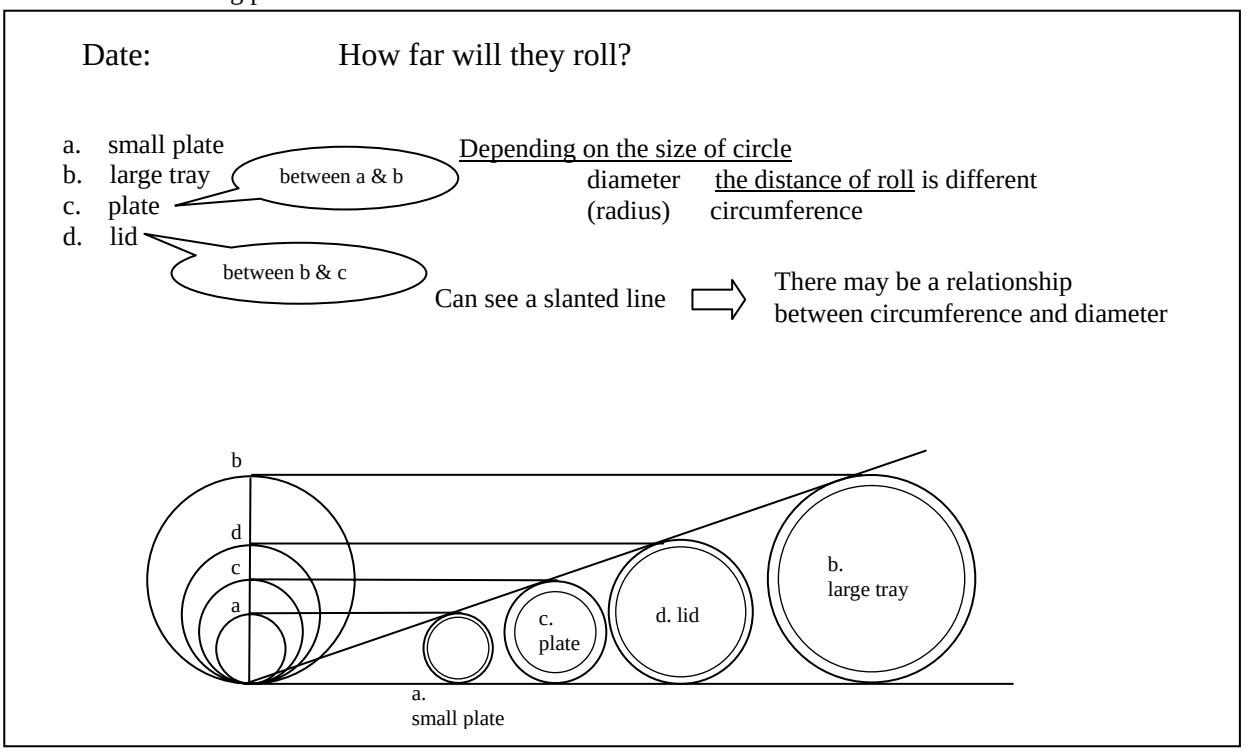
3. Instruction plan: Ratio of circumference and area of circles (8 lessons total)
Section 1: Ratio of circumference …. 4 lessons [today’s lesson is the first of 4]
Section 2: Area of circles … 4 lessons

4. Instruction in this lesson
(1) Goals
- Students will notice that there is a relationship between the circumference and the diameter through the activity of rolling circular objects.

(2) Flow of the lesson

Instructional Activity	Points of Considerations
1. Predict the length around a 1-yen coin. 3 cm 4 cm 5 cm 6 cm 2. Explore how far various circular objects will roll along the ledge of blackboard. When these objects are rotated once, how far will they go?    ・ When the diameter is fixed, the length around the circle is also fixed. ・ The larger the diameter, the further an object will roll. ・ The length around the object appears to be about 3 times of the diameter.	・ It is anticipated that most children will predict 3 or 4 cm. Acknowledge the surprise they feel when they find out that the length exceeds 6 cm and their desire to actually roll the coin to verify this fact. ・ Have students predict how far each object will roll first. ・ Use familiar objects such as trays, plates, and cans. ・ To help them recognize the dependency relationship more easily, alternate large and small objects. After each object, have students record what they noticed in their notebook. ・ After several objects are rolled, affirm that there is a relationship between the circumference and the diameter. ・ During the class discussion, use different colored chalks to identify clearly what have been discovered. ・ Present the formal term, “circumference” for the length around a circle. ・ Acknowledge the disposition to further investigate the relationship between the circumference and the diameter.

Blackboard writing plan



APEC Specialists Meeting

Mathematics Public Lesson I "Geometric Construction"

Date of the lesson: Wednesday, January 18, 2006
Teacher: Suzuki, Akihiro

1. Class Lower Secondary Grade 1 (Grade 7), Homeroom No. 4 (21 Boys, 20 Girls)
2. Unit Plane Figures
3. Goals of the lesson

This lesson addresses the following learning goal stated in the National Course of Study:

> *(1) To enhance students' ability to construct basic geometric figures with foresight while deepening their understanding of plane figures.*
>
> *b. To help students understand the basic geometric construction processes such as construction of angle bisector, perpendicular bisector of a segment, and perpendicular line to a given line.*

However, the Teaching Guide for the Course of Study further states,

> *Not only construction of geometric figures is a fundamental skill important in the study of geometric figures but also it serves the purpose of motivating students to become interested in the study of geometric figures, deepening their ways of observing and thinking, and facilitating logical examination of geometric figures.*

The goal of this lesson will include this development of mathematical ways of observing and thinking. In particular, the lesson is positioned as an opportunity to facilitate logical examinations of geometric figures.

Up to this point, through manipulation of concrete objects such as cutting or folding papers, students have studied the basic ideas of geometric figures and symmetries. By considering the question, "How can we think about the situation if manipulation of concrete objects is not possible?" they developed generalizations.

In teaching drawing of geometric figures, the focus of instruction shifts from actual manipulation such as cutting and folding to construction with compass and ruler. This transition involves not only a change in the tools of drawing but also a shift toward more abstract treatment and logical examination of geometric figures.

Therefore, in today's lesson, I would like students to understand the necessity for logical examination of geometric figures based on construction activities.

In today's lesson, we use a figure (called Landolt Ring) that is found in the chart used for vision examinations. This figure was previously used in the study of direct and indirect proportion. At that point, students actually measured various distances as well as cut and folded the figure. Today's lesson is built on those experiences.

4. Instruction Plan
   (1) Basics of plane figures .. 2 lessons
   (2) Symmetrical figures .. 4 lessons
   (3) Construction .. 4 lessons
   - Rules of construction, construction of perpendicular bisector, circles .. 2 lessons (today's lesson is the first of the two)
   - Construction of perpendicular lines, angle bisectors ……. 1 lesson
   - Other construction ……. 1 lesson

5. Flow of the lesson

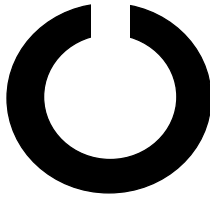
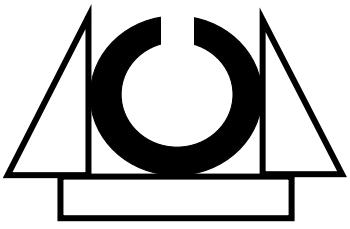
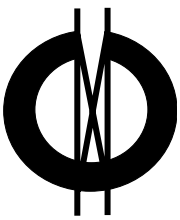
(1) Goals

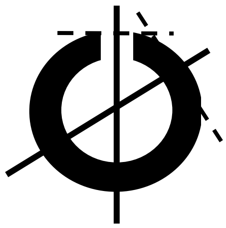
By determining the diameter of a Landolt Ring using a variety of methods, students will develop the procedure for constructing perpendicular bisector and examine rules of construction.

(2) Materials

Worksheet, compass, ruler, chart for the vision examination

(3) Steps of instruction

Time (min.)	Instructional activity	Points of consideration	Evaluation points and methods
5	・ Posing & understanding the task Display the examination chart. T: When we studied direct and indirect proportion, we determined the diameter of a Landolt Ring. At that point, you actually cut and folded the paper figures. In today's lesson, let's think about ways of determining the diameter without cutting and folding the paper model. Distribute the worksheet. Let's think about ways to determine the diameter of Landolt Ring without cutting and folding. 		
20	T: In your notebook, please record < Method> and <Why the method is correct> ・ Individual problem solving <Anticipated solutions:> (1) using rulers.  (2) drawing parallel lines: As a way to determine the center of point symmetry, find the point of intersection of a pair of segments connecting corresponding points. Then, measure the diameter. 	For students who cannot get started, ask: "What did you do when you were studying direct and indirect proportion?" "What does it mean that two sides match when you fold the paper model?" Students who are successful should think about multiple methods.	While circulating ・ Do they understand the task? ・ Are they engaged in problem solving? Evaluate through their written work .

	(3) draw perpendicular bisectors of chords then, using the point of intersection as the center, determine the diameter. 		
20	・ Whole class problem solving T: Before we start discussion, please review and revise <Method> and <Why the method is correct> you wrote in your notebook. Discussion (1) Have students share their <Method> and <Why the method is correct>, and critique each other's idea. (2) Identify both good and not-so-good points of each shared idea. (3) From the viewpoint of "accuracy," summarize those methods that can be considered as construction.	If some students cannot write their methods and the rationale, allow them to simply list some key terms. For those students who are more advanced, have them think about how to write their ideas so that others can more easily understand it.	While circulating ・Do students have their own ideas? ・Can they express their ideas using their own words? Evaluate by checking students writing in their notebooks. ● Were they able to identify the method of constructing perpendicular bisector? ● Were they able to think about the rules of construction? Evaluate by listening to students' comments during the discussion.
5	● Conclusion of the lesson (1) Rules of construction ● Ruler is used only to draw a line connecting 2 points ● Compass is used to draw either a circle or copy a length (2) Which methods shared in today's lesson can be considered as construction? [What was constructed?] (3) What were you able to do by constructing perpendicular bisector? [What were you able to determine?]		

APEC Specialists Meeting

Mathematics Public Lesson 2 "Volume of Tetrahedron with congruent faces"

Date of the lesson: Wednesday, January 18, 2006

Teacher: Ohneda, Yutaka

1. Class Lower Secondary Grade 3 (Grade 9), Homeroom No.4 (21Boys, 20Girls)
2. Introduction

Instead of viewing mathematics learning as a passive process of receiving mathematical knowledge as cultural products from the teacher to students, we should consider it as an active process with teacher support, where students possess the disposition, "Let's make new mathematics." Therefore, I believe it is important for teachers to continuously strive to identify factors that support the establishment of mathematical activities in a lesson that facilitate students' thinking.

It is important that students feel pleasant about the experiences of such mathematical activities. It is essential that mathematics lessons meet students' intrinsic needs. It is my hope that I can enjoy mathematics with students by developing lessons that will allow students to feel the "benefits and pleasure of mathematics,"

3. Perspective on instructional material

As a result of the reduction of the number of mathematics lessons in the current National Course of Study, the topic, cutting and projecting of solid figures, has been eliminated. However, as the phrases such as "enjoying mathematical activities," and "observing, manipulating and experimenting" suggest mathematical ways of observing and thinking reflected in cutting and projecting solid figures are absolutely necessary and must be taught regardless. Thus, although these topics have been eliminated from the Course of Study, teachers are still expected to provide the necessary instruction. I am very concerned about the status of such a foundational content for developing spatial sense.

For example, suppose you received a present in a little box. You will be very excited and wonder, "What is my present?" You will imagine various items that might be in the box. Based on the size, weight, color, sound, etc., you try to determine what might be in the box. If the present is a jewelry, you will look at it from a variety of angles and observe its shape, brightness, and design. When we try to recognize an object, we observe it from different angles and do a variety of things to analyze what it is made of and how it is shaped. In that process, the mathematical ways of observing and thinking reflected in cutting and projecting solid figures become necessary.

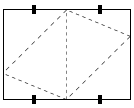
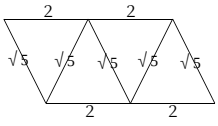
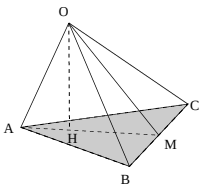
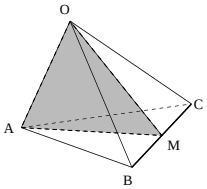
4. Goals of today's lesson and lesson plan

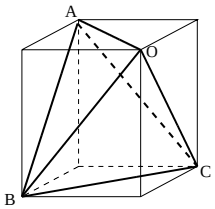
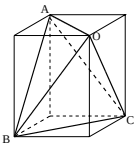
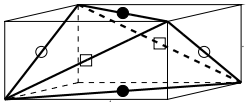
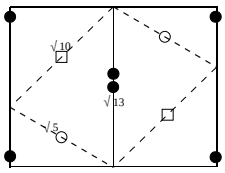
At our school, we continue to develop a geometry curriculum recognizing the important role of space in the domain of geometry. In the second grade of lower secondary school (Grade 8), we plan a series of lessons centered around the notion of projections of space figures. In the third grade (Grade 9), our lessons will explore measurements of space figures, incorporating various mathematical activities. At this point of the school year, we have already completed the required contents of the third grade (Grade 9); therefore, we plan lessons that are more topical as enrichment.

The first goal of today's lesson is to make students become interested in the exploration by folding papers to construct tetrahedron with congruent faces. Because the students will have the actual tetrahedron by their sides, they will be more easily imagine the process of measurement concretely, and mathematical activities will be facilitated. Furthermore, by thinking about solid tetrahedron with congruent faces, we attempt to shift the way students may be looking at the problem so that they can understand the way the solids are structured more deeply.

The emphasis here is to go beyond looking at geometric figures analytically and to deepen their examination of space figures further by considering the viewpoint of actually making the object.

5. Flow of the lesson

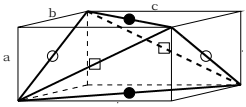
Content	Instructional Activities	Points of consideration & evaluation
Introduction of the task Actually fold the given design to confirm that the space figure is a tetrahedron with congruent faces.	Display the task on the blackboard. Examine what kind of space figure by actually folding the paper. <Task 1>> What kind of space figure will you have if you folded the design shown on the right?  By substituting numerical values, affirm that the faces are congruent isosceles triangles. → Understand the phrase, "Tetrahedron with congruent faces."	By having students engage in paper folding, promote interest in the task and facilitate mathematical activity.
Understanding the task Determining the volume of a tetrahedron with congruent faces.	<<Task 2>> Determine the volume of the tetrahedron with congruent faces you can construct from the figure shown on the right.  Anticipated students' solutions  We know that V = △ ABC × OH ÷3 but how can we calculate… → Any symple way? [Solution 1] △ ABC is an equilateral triangle with each side 2 unit long. Therefore, OH = √3 Consequently, V = △ ABC × OH ÷3 = (AM × BC ÷2) × √3÷3 =2√3／3 [Solution 2] If we cut the tetrahedron by plane OAM, we obtain two congruent triangular prisms. Therefore,  V = (△ OAM × BM ÷3) ×2 = △ OAM × BC ÷3 = √3×2÷3	By having an actual tetrahedron with congruent faces, students might be able to more easily have an image of the task, facilitating mathematical activity. To support students ' note taking and as a focus point for thinking, record this on the blackboard. To avoid careless computational error, make sure that students' writing is clear and easy to understand. → An important step toward high school mathematics.

	[Solution 3] The given tetrahedron with congruent faces can be obtained by cutting a square prism as shown in the figure below. Therefore, V = Volume of Cube ÷3 =(Area of Base × Height ÷3 =2×√3÷3  When we made a tetrahedron with congruent faces by folding paper, inside was empty. How can we make such a tetrahedron that is solid? Anticipated student response → We can cut a wooden square prism. So, … [Solution 2]  	If this solution is not proposed, proceed to the next task. Omit [Cutting of a cube] (below) if [Solution 3] is proposed.
Enrichment Task 1 Cutting of a cube	<<Task 3>> If we cut a cube (for example an erase), what kind of tetrahedron with congruent face can we make? Anticipated students response → We can get a tetrahedron with congruent faces that are scalene triangles. 	Limit the discussion to a brief explanation. To support students ' note taking and as a focus point for thinking, record this on the blackboard.
Enrichment Task 2 Constructing a model of tetrahedron with congruent faces	By folding arbitrary rectangular papers, make the tetrahedron with congruent faces shown above. <<Task 4>> Verify that you can make the given tetrahedron with congruent faces by folding an arbitrary rectangular paper.	Understand that the tetrahedron with congruent faces can be made by folding an paper.
Enrichment task 3 Calculating the volume of various tetrahedral with congruent faces	Provide different set of measurements and calculate the volume. <<Task 5>> Calculate the volume of a tetrahedron with the edges with lengths $\sqrt{5}$, $\sqrt{10}$, and $\sqrt{13}$. Anticipated students response: Let the lengths of the edges of the rectangular prism be *a*, *b*, and *c*. Then,  	Advanced students might simply calculate the volume, but make sure that their writing is clear and easy to understand.

$a^2 + b^2 = 5$

$b^2 + c^2 = 13$

$c^2 + a^2 = 10$



By solving this system of equations,

$a = 1, b = 2, c = 3$

Therefore,

$V = (1 \times 2 \times 3) \div 3$

$= 2$

Appendix

Related documents

List of Participants in Specialist Session

NAME	ECONOMY	AFFILIATION
Park Kyungmee	Korea	Hongik University
Catherine Lewis	USA	Mills College
Tad Watanabe	USA	Pennsylvania State University
Akihiko Takahashi	USA	DePaul University
Makoto Yoshida	USA	Global Education Resources
Max Stephens	Australia	University of Melbourne
Marsigit	Indonesia	The State University of Yogyakarta
Abdul Karim, Muchtar	Indonesia	State University of Malang
Lim Chap Sam	Malaysia	University of Science Malaysia
Soledad A. Ulep	Philippines	University of Philippines
Yeap Ban Har	Singapore	National Institute of Education Research
Suladda Loipha	Thailand	Khon Kaen University
Maitree Inprasitha	Thailand	Khon Kaen University
Supotch Chaiyasang	Thailand	Sri-Nakharinwirot University
Wichai Chumni	Thailand	Thaksin University
Pasaad Kongtaln	Thailand	Khon Kaen University
Pennee Narot	Thailand	Khon Kaen University
Chantana Klomjit	Thailand	Khon Kaen University
Auijit Pattanajak	Thailand	Khon Kaen University
Frederick Leung	Hong Kong	University of Hong Kong
Grecia Gálvez	Chile	Ministerio de Educación
Le Hai khoi	Vietnam	Director of the Institute of Information Technology
Tran Vui	Vietnam	Hue University
Masami Isoda	Japan	University of Tsukuba
Kazuyoshi Okubo	Japan	Hokkaido University of Education Sapporo
Masahiro Arimoto	Japan	National Institute for Educational Policy Research of Japan

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Committee	Kazuyoshi Okubo	Hokkaido University of Education
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	Minoru Yoshida	Shinshu University
	Noboru Saito	Naruto University of Education
	Takuya Baba	Hiroshima University
	Hiroyuki Ninomiya	Ehime University

Editors	Takeshi Miyakawa	University of Tsukuba
	Kazuhiro Aoyama	University of Tsukuba
	Kimiho Chino	University of Tsukuba

Symposium on International Cooperation

15 January 2006, PROGRAM

09:30 Opening Ceremony Chair: Masami Isoda (University of Tsukuba)

Kazuhisa Matsuoka, Director of JICA

Tadao Nakahara, President of Japan Society of Mathematical Education

Masakata Ogawa, President of Japan Society for Science Education

10:00 Keynote Address Chair: Shizumi Shimizu (University of Tsukuba)

"Mathematical Literacy for Living from OECD-PISA perspective"

Jan de Lange Director, Freudenthal Institute, Netherlands;
Chair, OECD-PISA technical committee

11:00 Keynote Address Chair: Hiroyuki Ninomiya (Ehime University)

"Japanese Lesson Study for Developing Best Practice"

Akihiko Takahashi (DePaul University, USA)

Shizumi Shimizu (University of Tsukuba)

12:00 Lunch Break

13:00 Panel for sharing the ideas of projects: planning international cooperation on Numeracy

Chair: Izumi Nishitani (Gunma University)

Kazuyoshi Okubo (Hokkaido University of Education)

Lecture:

"Status of Cooperation in Mathematics and Science Education by JICA"

Masazumi Ogawa (JICA)

Presentation: Project Report

South Africa Ronel Paulsen (University of South Africa)

Kenya Nancy Wambui Nui & Alice Nyacomba Wahome (SMASSE)

- 5 min questions -

- 10-min break -

Egypt Nasser Elsayed Abdel-hamed Ebada (NCERD)

Indonesia Muchtar Abdul karim (State University of Malang)

Honduras Jose Gerardo Fuentes (UPNFM)

- 15-min break for arrangement -

15:35 Panel discussion: Creation of "Good Lessons" in each country

Moderator: Takuya Baba (Hiroshima University)

Panelists: Presenters of project

16:55 Closing Ceremony Kazuyoshi Okubo (Hokkaido University of Education)

17:30 Social

筑波大学

APEC - TSUKUBA International Conference

Asia-Pacific Economic Cooperation

Innovative Teaching Mathematics through Lesson Study

DATE : January 15 ～ January 20, 2006
VENUE: Institute for International Cooperation
(10-5, Ichigaya Honmura-cho, Shinjuku-ku, Tokyo 162-8433)

Organized by : University of Tsukuba
Co-organized by : Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT)
Japan International Cooperation Agency (JICA)
Supported by : Ministry of Foreign Affairs of Japan (MOFA)
Japan Society of Mathematical Education (JSME)
Japan Society for Science Education (JSSE)

■Symposium on International Cooperation : January 15

"Improving the Quality of Education for Developing Numeracy on Education for All: Planning the International Collaboration for Future"

Keynote Addresses:
"Mathematical Literacy for Living from OECD-PISA perspective"
Dr. Jan de Lange, Director, Freudenthal Institute, Netherlands
Chair, OECD-PISA technical committee
"Japanese Lesson Study for Developing Best Practice"
Professor Akihiko Takahashi, DePaul University
Professor Shizuo Shimizu, University of Tsukuba

Panel for sharing the ideas of projects planning international collaboration on Numeracy:
"How have countries adopted the Lesson Study Approach for Educational Development on their JICA Projects ?"

■APEC Symposium January 16

"Innovative Teaching Mathematics through Lesson Study"
The Result from the past APEC Conference

Keynote Address:
"Professional Development through Lesson Study: A Lesson learned from US"
Professor Catherine C. Lewis, Mills College USA
"Comparative Study of Mathematics Classroom"
Professor Frederick Leung, Hong Kong

Lecture 1 "Innovation of mathematics teaching with ICT"
Professor Yoshiyuki Iijima, Aichi University of Education
Lecture 2 "Good Practice in Korea"
Professor Kyungwa Park, South Korea
Lecture 3 "Open-ended Approach and Teacher Education"
Professor Maitree Inprashita, Thailand
General Discussion:
"Developing Research for Good Practice and its Methods"

■APEC Conference : January 17 - January 20

January 17 Lesson Study for Mathematics
Attached Elementary School and University of Tsukuba at Tokyo
January 18 Lesson Study for Mathematics
Attached Junior High School and University of Tsukuba at Tokyo

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